The course is an introduction to symmetry analysis in fluid mechanics. The student will learn how to find similarity and travelling-wave solutions to partial differential equations used in fluid and continuum mechanics. The course covers mathematical and physical aspects

Content

**Chapter 1: The concept of similarity**
- Geometrical similarity
- Invariance by affine transformation, rotation, translation
- Fractal similarity
- Scaling law
- Physical similarity
- Complete similarity: drag force
- Incomplete similarity: flow resistance

**Chapter 2: Transport phenomena in fluid dynamics**
- Transport phenomena
- Advection
- Diffusion Heat equation
- Wave
- Shocks and conservation equations
- Boundary problems: fixed boundary, boundary layer, free boundary problem
- Classification of partial differential equations
- First-order equation, characteristic form
- Second order equation, hyperbolic, elliptic, parabolic

**Chapter 3: One-parameter groups, Lie groups**
- Groups of transformation
- Group invariants
- Invariant curves
- Transformation of derivative

**Chapter 4: First-order differential equations**
- Phase portrait
- Singular point
- Separatrix
- Integrating factor
- Invariant integral curves
- Singular solution
• Change of variables

Chapter 5: Second-order differential equations
• Invariant differential equations
• Lie’s reduction theorem
• Stretching group
• Singularities

Chapter 6: Similarity solutions to partial differential equation
• Similarity solutions
• Associated stretching group
• Asymptotic behavior
• Determining equations

Chapter 5: Travelling wave solution
• Translation groups
• Example: diffusion with source
• Propagation velocity
• Approach to travelling waves

Chapter 8: Hyperbolic problems
Hyperbolic problems
• One dimensional problems
• Characteristic equations
• Shock formation
• The Riemann problem
  Generalization to multidimensional problems
  • Linear systems
  • Nonlinear systems
  • The shallow-water equations

Chapter 9: Parabolic problems
• Linear diffusion
• Nonlinear diffusion
• Stefan problem
• Boundary layer problem

Keywords
partial differential equation, diffusion, advection, similarity solutions, travelling wave solution, hyperbolic problems

Resources
  Bibliography
  Bibliography is provided on the webpage

Websites
• http://lhe.epfl.ch/doctorate-en.php