Gödel and recursivity

Summary
Gödel incompleteness theorems and mathematical foundations of computer science

Content
Gödel's theorems:

Recursivity:

Keywords
Gödel, incompleteness theorems, Peano arithmetic, Robinson arithmetic, decidability, recursively enumerable, arithmetical hierarchy, Turing machine, Turing degrees, jump operator, primitive recursive functions, recursive functions, automata, pushdown automata, regular languages, context-free languages, recursive languages, halting problem, universal Turing machine, Church thesis.

Learning Prerequisites
Recommended courses
Mathematical logic (or equivalent). A good understanding of 1st order logic is required - in particular the relation between syntax and semantics.

Important concepts to start the course
1st order logic: syntax, semantics, proof theory, completeness theorem, compactness theorem, Löwenheim-Skolem theorem.

Learning Outcomes
By the end of the course, the student must be able to:
• Estimate whether a given theory, function, language is recursive or no
• Decide the class that a language belongs to (regular, context-free, recursive,...)
• Elaborate an automaton
• Design a Turing machine
• Formalize a proof in Peano arithmetic
• Sketch the incompleteness theorems
• Propose a non-standard model
• Argue why Hilbert program failed

Teaching methods
Ex cathedra lecture and exercises

Assessment methods
Written: 3 hours
Dans le cas de l’art. 3 al. 5 du Règlement de section, l’enseignant décide de la forme de l’examen qu’il communique aux étudiants concernés.

Supervision
Office hours: Yes
Assistants: Yes
Forum: Yes

Resources
Bibliography
Set Theory:
• Thomas Jech: Set theory, Springer 2006
• Kenneth Kunen: Set theory, Springer, 1983
• Jean-Louis Krivine: Theory des ensembles, 2007
• Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
• Yiannis Moschovakis: Notes on set theory, Springer 2006
• Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Recursion Theory :
• Piergiorgio Odifreddi: Classical recursion theory, vol. 1 and 2, Springer, 1999
• Robert I. Soare: Recursively Enumerable Sets and Degres, A Study of Computable Functions and Computably Generated Sets, Springer-Verlag 1987
• Nigel Cutland: Computability, an introduction to recursive function theory, 1980
• Raymond M. Smullyan: recursion theory for methamathematics, Oxford, 1993

Proof theory :
• Wolfram Pohlers: Proof Theory, the first step into impredicativity, Springer, 2008
• A. S. Troelstra, H. Schwichtenberg, and Anne S. Troelstra: Basic proof theory, Cambridge, 2000

Gödel's results :
• Raymond M. Smullyan: Gödel's incompleteness theorems, Oxford, 1992
• Peter Smith: An introduction to Gödel's theorems, Cambridge, 2008
Ressources en bibliothèque

- Théorie des ensembles / Krivine
- Introduction to Set theory / Hrbacek
- Proof Theory / Pohlers
- Notes on theory / Moschovakis
- Basic proof theory / Troelstra
- Introduction to the Theory of Computation / Sipser
- Handbook of proof theory / Buss
- Set theory / Jech
- Classical recursion theory / Odifredi
- Recursion theory for methamathematics / Smullyan
- Set theory / Kunen
- Incompleteness in the land of sets / Fitting
- Recursively Enumerable Sets and Degrees / Soare
- Gödel's theorem / Franzen
- Computability, an introduction to recursive function theory / Cutland
- Logique et théorie des ensembles / Dehornoy
- Gödel's incompleteness theorems / Smullyan
- An introduction to Gödel's theorems / Smith
- Inexhaustibility, a non exhaustive treatment / Franzen

Websites

- http://www.hec.unil.ch/logique/enseignement/recursivity

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