Remarque
Cours donnés en alternance tous les deux ans (pas donné en 2019-20)

Summary
Gödel incompleteness theorems and mathematical foundations of computer science

Content

Gödel's theorems:

Recursivity:

Keywords
Gödel, incompleteness theorems, Peano arithmetic, Robinson arithmetic, decidability, recursively enumerable, arithmetical hierarchy, Turing machine, Turing degrees, jump operator, primitive recursive functions, recursive functions, automata, pushdown automata, regular languages, context-free languages, recursive languages, halting problem, universal Turing machine, Church thesis.

Learning Prerequisites

Recommended courses
Mathematical logic (or equivalent). A good understanding of 1st order logic is required - in particular the relation between syntax and semantics.

Important concepts to start the course
1st order logic: syntax, semantics, proof theory, completeness theorem, compactness theorem, Löwenheim-Skolem theorem.

Learning Outcomes
By the end of the course, the student must be able to:
• Estimate whether a given theory, function, language is recursive or no
• Decide the class that a language belongs to (regular, context-free, recursive,...)
• Elaborate an automaton
• Design a Turing machine
• Formalize a proof in Peano arithmetic
• Sketch the incompleteness theorems
• Propose a non-standard model
• Argue why Hilbert program failed

Teaching methods
Ex cathedra lecture and exercises

Assessment methods
Written: 3 hours
Dans le cas de l’art. 3 al. 5 du Règlement de section, l’enseignant décide de la forme de l’examen qu’il communique aux étudiants concernés.

Supervision
Office hours Yes
Assistants Yes
Forum Yes

Resources
Bibliography
Set Theory:
• Thomas Jech: Set theory, Springer 2006
• Kenneth Kunen: Set theory, Springer, 1983
• Jean-Louis Krivine: Theory des ensembles, 2007
• Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
• Yiannis Moschovakis: Notes on set theory, Springer 2006
• Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Recursion Theory :
• Piergiorgio Odifreddi: Classical recursion theory, vol. 1 and 2, Springer, 1999
• Robert I. Soare: Recursively Enumerable Sets and Degres, A Study of Computable Functions and Computably Generated Sets, Springer-Verlag 1987
• Nigel Cutland: Computability, an introduction to recursive function theory, 1980
• Raymond M. Smullyan: recursion theory for methamathematics, Oxford, 1993

Proof theory :
• Wolfram Pohlers: Proof Theory, the first step into impredicativity, Springer, 2008
• A. S. Troelstra, H. Schwichtenberg, and Anne S. Troelstra: Basic proof theory, Cambridge, 2000

Gödel's results :
• Raymond M. Smullyan: Gödel's incompleteness theorems, Oxford, 1992
• Peter Smith: An introduction to Gödel's theorems, Cambridge, 2008
• Torkel Franzen: Inexhaustibility, a non exhaustive treatment, AK Peteres, 2002
• Melvin Fitting: Incompleteness in the land of sets, King's College, 2007
• Torkel Franzen: Gödel's theorem: an incomplete guide to its use and abuse, AK Peters, 2005

Ressources en bibliothèque
• Théorie des ensembles / Krivine
• Introduction to Set theory / Hrbacek
• Proof Theory / Pohlers
• Notes on theory / Moschovakis
• Basic proof theory / Troelstra
• Introduction to the Theory of Computation / Sipser
• Handbook of proof theory / Buss
• Set theory / Jech
• Classical recursion theory / Odifredi
• Recursion theory for methamathematics / Smullyan
• Set theory / Kunen
• Incompleteness in the land of sets / Fitting
• Recursively Enumerable Sets and Degrees / Soare
• Gödel's theorem / Franzen
• Computability, an introduction to recursive function theory / Cutland
• Logique et théorie des ensembles / Dehornoy
• Gödel's incompleteness theorems / Smullyan
• An introduction to Gödel's theorems / Smith
• Inexhaustibility, a non exhaustive treatment / Franzen

Websites
• http://www.hec.unil.ch/logique/enseignement/recursivity

Moodle Link
• http://moodle.epfl.ch/course/view.php?id=14569