Introduction to numerical modelling

Drezet Jean-Marie

Summary
This course aims to give a broad introduction to the basic numerical methods used to model physical phenomenon such as diffusion, heat transport, elasticity and the wave equation, and incompressible fluid dynamics. Computational examples will be given within the python framework.

Content
Recap on the ordinary differential equations (ODE) initial value problem
- the initial value (Cauchy) boundary condition
- numerical integration methods (Euler and Runge–Kutta, implicit trapezoidal)
- reduction of higher order ODEs to a system of 1st order ODEs
- truncation error, stability, stiffness, and propagation of error

The ODE boundary value problem
- General formulation of the ODE boundary value problem (Dirchlet/Neumann boundary conditions)
- numerical solution methods (shooting and finite difference)
- 1D solution examples

Partial differential equations (PDE)
- general definition of PDEs and their boundary conditions
- differential operators – their physical origin, continuity, flux conservation, material derivative
- development of well known examples (diffusion, heat and wave equation, the Navier-Stokes equation, and solid mechanics (elasticity)
- numerical solution methods (finite difference and finite element)

Keywords
Numerical methods, Ordinary Differential Equations, Partial Differential Equations, Finite difference methods, Finite element methods

Learning Prerequisites
Important concepts to start the course
- finite difference representation of derivatives
- numerical integration – quadrature (Newton-Coates)
• basic linear algebra – solution of linear systems
• numerical solution of non-linear equations

Learning Outcomes
By the end of the course, the student must be able to:
• Model some common physical phenomenon
• Propose the appropriate numerical solution strategy for a variety of different physical models

Transversal skills
• Use a work methodology appropriate to the task.
• Communicate effectively with professionals from other disciplines.
• Demonstrate the capacity for critical thinking
• Take feedback (critique) and respond in an appropriate manner.

Teaching methods
A weekly lecture will be given covering all theoretical concepts. Included in these lectures will be numerical solution examples using python

Expected student activities
Attendance to lectures

Assessment methods
Assessment is via an oral exam