TNT: Automorphic forms and L-functions

Summary
Modular forms have a central place in number theory and occur also in many other branches of mathematics. Starting from the theta series associated to quadratic forms we will introduce the basic concepts associated to modular or automorphic forms and provide some applications of the theory.

Content
1. Spherical harmonics, quadratic forms and theta functions
2. The space of lattices, SL_2, the Poincaré upper-half plane and its arithmetic quotients
3. The space of holomorphic modular forms. Fourier expansion, Petersson inner product and the Petersson formula.
4. Hecke theory and L-functions
5. Applications: integral points on spheres, design of golden quantum gates, construction of Ramanujan graphs (Pizer and Lubotsky-Phillips-Sarnak)

Keywords
lattices, modular forms, L-functions, quadratic forms.

Learning Prerequisites
Required courses
Introduction to analytic number theory,
Algebraic number theory,
Analysis III,
Analysis IV

Learning Outcomes
By the end of the course, the student must be able to:
• Synthesize the theory of modular forms and their applications
• Solve basic problems involving modular forms

Teaching methods
course ex-cathedra
exercises
**Expected student activities**
proactive attitude during the courses and the exercises sessions (possibly with individual presentation of the solution of various problems).

**Assessment methods**
oral presentation

**Supervision**

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