Homotopical algebra

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<tr>
<th>Cursus</th>
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<td>Ing.-math</td>
<td>MA2, MA4</td>
<td>Opt.</td>
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<td>Mathématicien</td>
<td>MA2</td>
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**Summary**

This course will provide an introduction to model category theory, which is an abstract framework for generalizing homotopy theory beyond topological spaces and continuous maps. We will study numerous examples of model categories and their applications in algebra and topology.

**Content**

1. Category theory
2. Model categories and their homotopy categories
3. Transfer theorems
4. Localizing model categories
5. Monoidal model categories and "brave new algebra"

**Keywords**

Abstract homotopy theory

**Learning Prerequisites**

**Required courses**

Second-year math courses, including Topology.

**Recommended courses**

- Rings and modules
- Algebraic topology

**Important concepts to start the course**

- Necessary concept: homotopy of continuous maps
- Recommended concept: chain homotopy of morphisms between chain complexes

**Learning Outcomes**

By the end of the course, the student must be able to:
• Prove results in category theory involving (co)limits, adjunctions, and Kan extensions
• Prove basic properties of model categories
• Check the model category axioms in important examples
• Apply transfer theorems to establish the existence of model category structures
• Apply Bousfield localization to create model categories with desired weak equivalences
• Compare different model category structures via Quillen pairs
• Transpose results from classical algebra into homotopy-theoretic versions in monoidal model categories
• Check the axioms of a monoidal model category in important cases

Transversal skills
• Demonstrate a capacity for creativity.
• Demonstrate the capacity for critical thinking
• Continue to work through difficulties or initial failure to find optimal solutions.

Teaching methods
Ex-cathedra lectures, exercises

Expected student activities
Handing in weekly exercises to be graded.

Assessment methods
Written exam
Dans le cas de l’art. 3 al. 5 du Règlement de section, l’enseignant décide de la forme de l’examen qu’il communique aux étudiants concernés.

Supervision
Office hours No
Assistants Yes
Forum No

Resources
Bibliography


Ressources en bibliothèque

• (electronic version)
• (electronic version)
• Handbook of Algebraic Topology / James
• (electronic version)
• Simplicial Homotopy Theory / Goerss & Jardine
• Categorical Homotopy Theory / Riehl
• Model Categories / Hovey
• (electronic version)

Websites

• http://hessbellwald-lab.epfl.ch/HtpyAlg17