Summary

Algebraic K-theory, which to any ring R associates a sequence of groups, can be viewed as a theory of linear algebra over an arbitrary ring. We will study in detail the first two of these groups and applications of algebraic K-theory to number theory, algebraic topology, and representation theory.

Content

1. \( K_0 \): Grothendieck groups, stability, tensor products, change of rings, the Dévissage, Resolution and Localization theorems and their applications
2. \( K_1 \): elementary matrices, commutators and determinants, long exact sequences relating \( K_0 \) and \( K_1 \)

Keywords

Rings and modules, Grothendieck group

Learning Prerequisites

Required courses
Second-year algebra and topology courses

Recommended courses
Rings and modules (Anneaux et modules)

Important concepts to start the course
Elementary ring and field theory

Learning Outcomes

By the end of the course, the student must be able to:

- Compute group completions of various semi-groups
- Interpret the universal properties of group completions, Grothendieck groups, and universal determinants
- Compute the Grothendieck group of important subcategories of modules
- Apply the Dévissage, Resolution and Localization theorems
- Sketch the proofs of the Dévissage, Resolution, and Localization theorems
- Explain the functoriality of \( K_0 \)
- Compare the Grothendieck-type and matrix-based approaches to defining \( K_1 \)
• Prove elementary properties of $K_1$

Transversal skills

• Assess one's own level of skill acquisition, and plan their on-going learning goals.
• Continue to work through difficulties or initial failure to find optimal solutions.
• Demonstrate a capacity for creativity.

Assessment methods

Each student must hand in one exercise each week for correction, which will determine 30% of the final grade. The student's performance on the final written exam will determine the other 70% of the grade.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Resources

Websites

• http://gr-he.epfl.ch/AlgKthy20