Set theory

Set Theory as a foundational system for mathematics. ZF, ZFC and ZF with atoms. Relative consistency of the Axiom of Choice, the Continuum Hypothesis, the reals as a countable union of countable sets, the existence of a countable family of pairs without any choice function.

Content


Keywords

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe, L, Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

Learning Prerequisites

Required courses
MATH-381 Mathematical Logic.
In particular ordinal numbers and ordinal arithmetic will be considered known and admitted.

Recommended courses
Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get lost sooner or later.

Important concepts to start the course
• 1st order logic
• basics of proof theory
• Basics of model theory
• Compacity theorem
• Löwenheim-Skolem
• Completeness theorem

Learning Outcomes
By the end of the course, the student must be able to:
• Specify a model of ZFC
• Prove consistency results
• Develop a generic extension
• Argue by transfinite induction
• Decide whether ZFC proves its own consistency
• Formalize the axioms of ZF, AC, CH, DC
• Sketch an inner model
• Justify the axiom of foundation

Teaching methods
Ex cathedra lecture and exercises

Expected student activities
• Attendance at lectures
• Solve the exercises

Assessment methods
• Written exam (3 hours)
• Dans le cas de l’art. 3 al. 5 du Règlement de section, l’enseignant décide de la forme de l’examen qu’il communique aux étudiants concernés

Supervision
Office hours  Yes
Assistants  Yes
Forum  Yes

Resources
Virtual desktop infrastructure (VDI)
No

Bibliography
2. Lorenz Halbeisen: Combinatorial Set Theory, Springer 2018
5. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html

Ressources en bibliothèque
- Introduction to Set theory / Hrbacek
- Set theory / Jech
- Théorie des ensembles / Krivine
- Set theory / Kunen
- Notes on set theory / Moschovakis
- Logique et théorie des ensembles / Dehorny

Notes/Handbook
Lecture notes (350 pages).

Websites
- http://www.hec.unil.ch/logique/

Moodle Link