EPFL

MATH-318   **Set theory**

Duparc Jacques

Cursus	Sem.	Type
Computer science	MA1, MA3	Opt.
Ing.-math	MA1, MA3	Opt.
Mathematics for teaching	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.
SC master EPFL	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Set Theory as a foundational system for mathematics. Relative consistency of the Axiom of Choice and the Continuum Hypothesis.

Content

Set Theory: ZFC. Extensionality and Comprehension. Relations, functions, and well-ordering. Ordinals. Class and transfinite recursion. Cardinals. Well-founded relations, Axiom of foundation, induction, and von Neumann's hierarchy. Relativization, absoluteness, reflection theorems. Gödel's constructible universe L. Axiom of Choice, and Continuum Hypothesis inside L. Po-sets, filters and generic extensions. Forcing. ZFC in generic extensions. Cohen Forcing. Independence of the Continuum Hypothesis. HOD and the Axiom of Choice: independence of the Axiom of Choice.

Keywords

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe, L, Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

Learning Prerequisites

Recommended courses

Mathematical logic (or equivalent)

Important concepts to start the course

- 1st order logic
- basics of proof theory
- Basics of model theory
- Compacity theorem
- Löwenheim-Skolem

Learning Outcomes

By the end of the course, the student must be able to:

- Specify a model of ZFC
- Prove consistency results
- Develop a generic extension

- Argue by transfinite induction
- Decide whether ZFC proves its own consistency
- Formalize the axioms of ZF, AC, CH, DC
- Sketch an inner model
- Justify the axiom of foundation

Teaching methods

Ex cathedra lecture and exercises

Expected student activities

- Attendance at lectures
- Completing exercices

Assessment methods

- Writen exam (3 hours)

Supervision

Office hours	Yes
Assistants	Yes
Forum	No

Resources

Bibliography

1. Thomas Jech: Set theory, Springer 2006
2. Kenneth Kunen: Set theory, Springer, 1983
3. Jean-Louis Krivine: Theorie des ensembles, 2007
4. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
5. Yiannis Moschovakis: Notes on set theory, Springer 2006
6. Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Ressources en bibliothèque

- Introduction to Set theory / Hrbacek
- Set theory / Jech
- Logique et théorie des ensembles / Dehorny
- Set theory / Kunen
- Notes on set theory / Moschovakis
- Theorie des ensembles / Krivine

Websites

- http://www.hec.unil.ch/logique/enseignement