EPFL

MATH-501   **L-functions and random matrices**

Michel Philippe

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Obl.
Mathematics for teaching	MA2, MA4	Obl.
Mathématicien	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Remark

pas donné en 2017-18

Summary

This year, the course will discuss the analytic theory of L-functions. These objects of analytic nature (the epitome is Riemann's zeta function) encode deep arithmetic properties of number theoretic objects. We will discuss various modern methods to study them either individually or in families.

Content

I. Examples and basic properties.
II. Weil's explicit formula, zero free regions and prime number type theorems.
III. The distribution of zeros of L-functions: the Katz-Sarnak philosophy.
IV. L-functions inside the critical strip
- zero density estimates
- The mollification method
- the resonnance method.
- the amplification method.

Learning Prerequisites

Recommended courses

Analysis I II III IV, Algebra I II; Introduction to analytic number theory.

Important concepts to start the course

Some knowledge (and interest) for number theory along with advanced cerebral activity may be indispensable to start the course

Learning Outcomes

By the end of the course, the student must be able to:

• Demonstrate a mastery of the basic techniques studied in the course and during the exercise sessions.

• Synthesize more advanced techniques.

Teaching methods

Ex cathedra lectures.
The exercise sessions will be devoted to the understanding of the notions developed in the course and their extensions

to more general situations.

Expected student activities

A very active participation to the exercise sessions will be expected.

Assessment methods

Oral exam based on the material developed during the course and the exercise sessions.

Resources

Bibliography

E. Kowalski, H. Iwaniec, Analytic Number Theory.
J.-P. Serre, A Course in Arithmetic.

Ressources en bibliothèque

- Topics in Classical Automorphic Forms / Iwaniec
- A Course in Arithmetic / Serre

Prerequisite for

solving the Riemann hypothesis and winning the 1M USD Millenium prizes of the Clay Mathematics Institute