MATH-417

Topics in number theory

Michel Philippe

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Obl.
Mathematics for teaching	MA2, MA4	Opt.
Mathématicien	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

The course will cover the theory of algebraic exponential sums: sums over algebraic varieties over finite fields of algebraic functions. Emphasis will be put on applications to analytic number theory.

Content

The topic of this year's course is the theory of algebraic exponential sums:sums over algebraic varieties over finite fields of algebraic functions (composed with an additive or multiplicative character of the base field).
The basic example of these are Gauss sums (of prime modulus) and further example arose with the works of Hasse and Kloosterman. Andre Weil made a major advance to the theory in the case of one dimensional sums by solving the "Riemann hypothesis for curves" and forulated far reaching conjectures in the higher dimensional case which motivated Grothendieck definition of etale cohomology. The Weil conjectures were finally proven by Deligne in the 70's earning him the Fields medal.
The theory was further developped by Deligne, Laumon and Katz.
We will present an overview of the theory and will introduce the basic vocabulary. excepted for the case of case there will be very few proofs. Instead we will explain how to use these methods and apply them in problems coming from analytic number theory.
-Basic example of exponential sums.
-Exponential sums over a curve: a proof of the Riemann hypothesis for curves via Stepanov method.
-Overview of the Weil's conjectures, Grothendieck and Deligne's work.
-Basic notions and vocabulary concerning l-adic sheaves over a curve. Overview of the work of Katz and Laumon on the l-adic Fourier transform.
-Examples: Kloosterman sheaves and their associated trace functions
-Applications to analytic number theory : arithmetic functions in arithmetic progressions to large modulus, trace functions over the primes.

Learning Prerequisites

Required courses

Introduction a la Theorie Analytique des Nombres
Introduction a la Theorie Algebrique des Nombres

Recommended courses

Introduction to Algebraic Geometry

Learning Outcomes

By the end of the course, the student must be able to:

- Explain the content of the course
- Use the results presented to solve new problems in analytic number theory
- Work out / Determine by himself some specific example of algebraic exponential sums

Teaching methods

course ex-cathedra supplemented by exercices

Assessment methods

Oral examination.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes

Resources

Bibliography

Katz, Sommes d'exponentielles (Cours a Orsay) Asterique.
Katz, Gauss sums Kloosterman sums and monodromy, PUP

Ressources en bibliothèque

- Gauss sums Kloosterman sums and monodromy / Katz
- Sommes exponentielles / Katz

Notes/Handbook

The course notes will be made available on the moodle

Websites

- http://moodle.epfl.ch/course/view.php?id=15095