

MATH-468

**Numerical methods for saddle point problems**

Buffa Annalisa

| Cursus                                | Sem.     | Type |
|---------------------------------------|----------|------|
| Computational science and Engineering | MA2, MA4 | Opt. |
| Ing.-math                             | MA2, MA4 | Opt. |
| Mathematics for teaching              | MA2, MA4 | Opt. |
| Mathématicien                         | MA2, MA4 | Opt. |

|                            |                 |
|----------------------------|-----------------|
| Language of teaching       | English         |
| Credits                    | 5               |
| Session                    | Summer          |
| Semester                   | Spring          |
| Exam                       | Oral            |
| Workload                   | 150h            |
| Weeks                      | 14              |
| <b>Hours</b>               | <b>4 weekly</b> |
| Courses                    | 2 weekly        |
| Exercises                  | 2 weekly        |
| <b>Number of positions</b> |                 |

**Summary**

The aim of the course is to give a theoretical and practical knowledge of the finite element method for saddle point problems.

**Content**

- Minimization of convex functionals (energies) under linear constraints and their interpretation as saddle point problems. Wellposedness and inf-sup conditions.
- Finite element approximation of saddle point problems, discrete inf-sup conditions, stability and approximation estimates
- Finite elements for Stokes flows, (quasi-)incompressible linear elasticity, and Darcy flows
- Compatible discretisations of differential forms and of Maxwell equations

**Keywords**

Finite element methods, Galerkin approximation, mixed finite elements, Darcy flows, incompressible fluids and linear elasticity, Maxwell equations, discrete differential forms.

**Learning Prerequisites****Required courses**

Analysis I II III IV, Numerical Analysis, Advanced numerical analysis, Sobolev spaces and elliptic equations.

**Recommended courses**

Functional analysis I, measure and integration, Programming

**Important concepts to start the course**

- Basic knowledge of functional analysis, Banach and Hilbert spaces,  $L^p$  spaces
- Some knowledge on the theory of elliptic PDEs, weak solutions, existence and uniqueness
- Basic concepts in numerical analysis: stability, convergence, condition number, solution of linear systems, quadrature formulae, polynomial interpolation.

**Learning Outcomes**

By the end of the course, the student must be able to:

- Choose an appropriate discretisation scheme to solve a specific PDEs
- Analyse numerical errors
- Interpret results of a computation in light of theory
- Prove theoretical properties of discretisation schemes
- Propose a theoretical and numerical solution to a mini-project on a topic going beyond the material of the course
- Formalise the solution of a mini-project in a scientific report

### Transversal skills

- Use a work methodology appropriate to the task.
- Write a scientific or technical report.
- Use both general and domain specific IT resources and tools

### Teaching methods

Ex cathedra lectures, exercises in the classroom and computer lab sessions

### Expected student activities

- Attendance of lectures
- Completing exercises
- Solving problems on the computer
- Work out a small project and write a technical report

### Assessment methods

Written exams and evaluation of the report of a mini-project.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

|              |     |
|--------------|-----|
| Office hours | No  |
| Assistants   | Yes |

### Resources

#### Bibliography

- D. Boffi, F. Brezzi, M. Fortin Mixed Finite Element Methods and Applications, Springer Series in Computational mathematics, 2013.
- P. Monk, Finite Element Methods for Maxwell Equations, Oxford University press, 2003
- A. Ern, J-L. Guermond, Theory and Practise of Finite Elements, Springer 2004.

#### Ressources en bibliothèque

- [Mixed Finite Element Methods and Applications / Boffi & al.](#)
- [\(electronic version\)](#)
- [Theory and Practise of Finite Elements / Ern & Guermond](#)

#### Moodle Link

- <http://moodle.epfl.ch>