

MATH-318

**Set theory**

Duparc Jacques

Cursus	Sem.	Type
Computer science	MA2	Opt.
Cybersecurity	MA2	Obl.
Ing.-math	MA2, MA4	Opt.
Mathematics for teaching	MA2, MA4	Opt.
Mathématicien	MA2	Opt.
SC master EPFL	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Courses	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Remark**

Cours donné en alternance tous les deux ans (donné en 2018-19)

**Summary**

Set Theory as a foundational system for mathematics. Relative consistency of the Axiom of Choice and the Continuum Hypothesis.

**Content**

Set Theory: ZFC. Extensionality and Comprehension. Relations, functions, and well-ordering. Ordinals. Class and transfinite recursion. Cardinals. Well-founded relations, Axiom of foundation, induction, and von Neumann's hierarchy. Relativization, absoluteness, reflection theorems. Gödel's constructible universe  $L$ . Axiom of Choice, and Continuum Hypothesis inside  $L$ . Po-sets, filters and generic extensions. Forcing. ZFC in generic extensions. Cohen Forcing. Independence of the Continuum Hypothesis. HOD and the Axiom of Choice: independence of the Axiom of Choice.

**Keywords**

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe,  $L$ , Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

**Learning Prerequisites****Recommended courses**

Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get lost sooner or later.

**Important concepts to start the course**

- 1st order logic
- basics of proof theory
- Basics of model theory
- Compactness theorem
- Löwenheim-Skolem
- Completeness theorem

**Learning Outcomes**

By the end of the course, the student must be able to:

- Specify a model of ZFC
- Prove consistency results
- Develop a generic extension
- Argue by transfinite induction
- Decide whether ZFC proves its own consistency
- Formalize the axioms of ZF, AC, CH, DC
- Sketch an inner model
- Justify the axiom of foundation

### Teaching methods

Ex cathedra lecture and exercises

### Expected student activities

- Attendance at lectures
- Solve the exercises

### Assessment methods

- Written exam (3 hours)
- Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés

### Supervision

Office hours	Yes
Assistants	Yes
Forum	Yes

### Resources

#### Bibliography

1. Thomas Jech: Set theory, Springer 2006
2. Kenneth Kunen: Set theory, Springer, 1983
3. Jean-Louis Krivine: Theorie des ensembles, 2007
4. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: <http://www.math.unicaen.fr/~dehornoy/surveys.html>
5. Yiannis Moschovakis: Notes on set theory, Springer 2006
6. Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

#### Ressources en bibliothèque

- [Introduction to Set theory / Hrbacek](#)
- [Set theory / Jech](#)
- [Theorie des ensembles / Krivine](#)
- [Set theory / Kunen](#)
- [Notes on set theory / Moschovakis](#)
- [Logique et théorie des ensembles / Dehornoy](#)

#### Websites

- <http://www.hec.unil.ch/logique/>

**Moodle Link**

- <http://moodle.epfl.ch/course/index.php?categoryid=72>