EPFL

MATH-468

Numerical methods for saddle point problems

Buffa Annalisa

Cursus	Sem.	Type
Computational science and Engineering	MA2, MA4	Opt.
Ing.-math	MA2, MA4	Opt.
Mathematics for teaching	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

The aim of the course is to give a theoretical and practical knowledge of the finite element method for saddle point problems, such as fluid dynamics, elasticity and electromagnetic problems.

Content

- Minimization of convex functionals (energies) under linear constraints and their interpretation as saddle point problems. Wellposedness and inf-sup conditions.
- Finite element approximation of saddle point problems, discrete inf-sup conditions, stability and approximation estimates
- Finite elements for Stokes flows, (quasi-)incompressible linear elasticity, and Darcy flows
- Compatible discretisations of differential forms and of Maxwell equations

Keywords

Finite element methods, Galerkin approximation, mixed finite elements, Darcy flows, incompressible fluids and linear elasticity, Maxwell equations, discrete differential forms.

Learning Prerequisites

Required courses

Analysis I II III IV, Numerical Analysis, Advanced numerical analysis, Sobolev spaces and elliptic equations, Numerical Approximations of PDEs I

Recommended courses

Functional analysis I, measure and integration, Programming

Important concepts to start the course

- Basic knowledge of functional analysis, Banach and Hilbert spaces, L^p spaces
- Some knowledge on the theory of elliptic PDEs, weak solutions, existence and uniqueness
- Basic concepts in numerical analysis: stability, convergence, condition number, solution of linear systems, quadrature formulae, polynomial interpolation.

Learning Outcomes

By the end of the course, the student must be able to:

- Choose an appropriate discretisation scheme to solve a specific PDEs
- Analyse numerical errors
- Interpret results of a computation in light of theory
- Prove theoretical properties of discretisation schemes
- Propose a theoretical and numerical solution to a mini-project on a topic going beyond the material of the course
- Formalise the solution of a mini-project in a scientific report

Transversal skills

- Use a work methodology appropriate to the task.
- Write a scientific or technical report.
- Use both general and domain specific IT resources and tools

Teaching methods

Ex cathedra lectures, exercises in the classroom and computer lab sessions

Expected student activities

- Attendance of lectures
- Completing exercises
- Solving problems on the computer
- Work out a small project and write a technical report

Assessment methods

Oral exams and evaluation of the report of a mini-project.

Supervision

Office hours	No
Assistants	Yes

Resources

Bibliography

- D. Boffi, F. Brezzi, M. Fortin Mixed Finite Element Methods and Applications, Springer Series in Computatioanl mathematics, 2013.
- P. Monk, Finite Element Methods for Maxwell Equations, Oxford University press, 2003
- A. Ern, J-L. Guermond, Theory and Practise of Finite Elements, Springer 2004.

Ressources en bibliothèque

- Theory and Practise of Finite Elements / Ern & Guermond
- Mixed Finite Element Methods and Applications / Boffi & al.
- (electronic version)

Notes/Handbook

Notes for each lectures will be provided every week.

Moodle Link

- http://moodle.epfl.ch