

MATH-451

**Numerical approximation of PDE's I**

Nobile Fabio

Cursus	Sem.	Type
Computational science and Engineering	MA2, MA4	Opt.
Financial engineering	MA2, MA4	Opt.
Mathematics	BA6	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Courses	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Summary**

The aim of the course is to give a theoretical and practical knowledge of finite difference and finite element methods for the numerical approximation of partial differential equations in one or more dimensions.

**Content**

- Finite difference methods for elliptic, parabolic and hyperbolic equations; stability and convergence analysis; implementation aspects
- Linear elliptic problems: weak form, well-posedness, Galerkin approximation
- Finite element approximation in one, two and three dimensions: stability, convergence, a-priori error estimates in different norms, implementation aspects

**Keywords**

Partial Differential Equations, Finite difference method, Finite element method, Galerkin approximation, convergence analysis.

**Learning Prerequisites****Required courses**

Analysis I-II-III-IV, Numerical analysis

**Recommended courses**

Functional Analysis I, Measure and Integration, Espaces de Sobolev et équations elliptiques, Advanced numerical analysis, Programming

**Important concepts to start the course**

- Basic knowledge of functional analysis, Banach and Hilbert spaces,  $L^p$  spaces.
- Some knowledge on theory of PDEs, classical and weak solutions, existence and uniqueness.
- Basic concepts in numerical analysis: stability, convergence, condition number, solution of linear systems, quadrature formulae, finite difference formulae, polynomial interpolation.

**Learning Outcomes**

By the end of the course, the student must be able to:

- Choose an appropriate discretization scheme to solve a specific PDE
- Analyze numerical errors
- Interpret results of a computation in the light of theory
- Prove theoretical properties of discretization schemes
- Solve a PDE using available software
- State theoretical properties of PDEs and corresponding discretization schemes
- Describe discretization methods for PDEs

### Transversal skills

- Use a work methodology appropriate to the task.
- Use both general and domain specific IT resources and tools
- Write a scientific or technical report.

### Teaching methods

Ex cathedra lectures, exercises in the classroom and computer lab sessions

### Expected student activities

- Attendance of lectures
- Completing exercises
- Solving simple problems on the computer

### Assessment methods

written exam. The exam may involve the use of a computer.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

Office hours	Yes
Assistants	Yes
Forum	No

### Resources

#### Virtual desktop infrastructure (VDI)

No

### Bibliography

- D.F. Griffiths, J.W. Dold, D.J. Silvester, *Essential partial differential equations*. Springer 2015.
- S. Larsson, V. Thomée, *Partial differential equations with numerical methods* (Vol. 45). Springer Science & Business Media, 2008
- A. Quarteroni, *Numerical Models for Differential Problems*, Springer, 2009
- S.C. Brenner, L.R. Scott *The Mathematical Theory of Finite Element Methods*, Springer, 3rd ed, 2007
- A. Ern, J-L. Guermond, *Theory and Practice of Finite Elements*, Springer, 2004
- Lecture notes by the teacher

### Ressources en bibliothèque

- Numerical Models for Differential Problem / Quarteroni
- Theory and Practice of Finite Elements / Guermond
- (version électronique)
- (version électronique)
- Partial differential equations with numerical methods / Larsson & Thomée
- The Mathematical Theory of Finite Element Methods / Brenner & Scott
- Essential partial differential equations / Griffiths & al.
- (version électronique)
- (version électronique)

**Moodle Link**

- <http://moodle.epfl.ch/>

**Prerequisite for**

Numerical Approximation of Partial Differential Equations II, Numerical methods for conservation laws, Numerical methods for saddle point problems