MATH-483 **Gödel and recursivity**

Cursus	Sem.	Type
Computer science	MA1, MA3	Opt.
Cybersecurity	MA1, MA3	Opt.
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.
SC master EPFL	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

Cours donnés en alternance tous les deux ans (pas donné en 2019-20)

Summary

Gödel incompleteness theorems and mathematical foundations of computer science

Content

Gödel's theorems:
Peano and Robinson Arithmetics. Representable functions. Arithmetic of syntax. Incompleteness, and undecidability theorems.

Recursivity :
Turing Machines and variants. The Church-Turing Thesis. Universal Turing Machine. Undecidable problems (the halting and the Post-Correspondance problems). Reducibility. The arithmetical hierarchy. Relations to Turing machines. Turing degrees.

Keywords

Gödel, incompleteness theorems, Peano arithmetic, Robinson arithmetic, decidability, recursively enumarable, arithmetical hierarchy, Turing machine, Turing degrees, jump operator, primitive recursive functions, recursive functions, automata, pushdown automata, regular languages, context-free languages, recursive languages, halting problem, universal Turing machine, Church thesis.

Learning Prerequisites

Recommended courses

Mathematical logic (or equivalent). A good understanding of 1st order logic is required - in particular the relation between syntax and semantics.

Important concepts to start the course

1st order logic: syntax, semantics, proof theory, completeness theorem, compactness theorem, Löwenheim-Skolem theorem.

Learning Outcomes

By the end of the course, the student must be able to:

- Estimate whether a given theory, function, language is recursive or no
- Decide the class that a language belongs to (regular, context-free, recursive,...)

- Elaborate an automaton
- Design a Turing machine
- Formalize a proof in Peano arithmetic
- Sketch the incompleteness theorems
- Propose a non-standard model
- Argue why Hilbert program failed

Teaching methods

Ex cathedra lecture and exercises

Assessment methods

Written: 3 hours
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	Yes
Assistants	Yes
Forum	Yes

Resources

Bibliography

Set Theory:

- Thomas Jech: Set theory, Springer 2006
- Kenneth Kunen: Set theory, Springer, 1983
- Jean-Louis Krivine: Theory des ensembles, 2007
- Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
- Yiannis Moschovakis: Notes on set theory, Springer 2006
- Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Recursion Theory :

- Micheal Sipser: Introduction to the Theory of Computation, Thomson Course Technology Boston, 2006
- Piergiorgio Odifreddi: Classical recursion theory, vol. 1 and 2, Springer, 1999
- Robert I. Soare: Recursively Enumerable Sets and Degres, A Study of Computable Functions and Computably Generated Sets, Springer-Verlag 1987
- Nigel Cutland: Computability, an introduction to recursive function theory, 1980
- Raymond M. Smullyan: recursion theory for methamathematics, Oxford, 1993

Proof theory :

- Wolfram Pohlers: Proof Theory, the first step into impredicativity, Springer, 2008
- A. S. Troelstra, H. Schwichtenberg, and Anne S. Troelstra: Basic proof theory, Cambridge, 2000
- S.R. Buss: Handbook of proof theory, Springer, 1998

Gödel's results :

- Raymond M. Smullyan: Gödel's incompleteness theorems, Oxford, 1992
- Peter Smith: An introduction to Gödel's theorems, Cambridge, 2008

- Torkel Franzen: Inexhaustibility, a non exhaustive treatment, AK Peteres, 2002
- Melvin Fitting: Incompleteness in the land of sets, King's College, 2007
- Torkel Franzen: Gödel's theorem: an incomplete guide to its use and abuse, AK Peters, 2005

Ressources en bibliothèque

- Théorie des ensembles / Krivine
- Introduction to Set theory / Hrbacek
- Proof Theory / Pohlers
- Notes on theory / Moschovakis
- Basic proof theory / Troelstra
- Introduction to the Theory of Computation / Sipser
- Handbook of proof theory / Buss
- Set theory / Jech
- Classical recursion theory / Odifreddi
- Recursion theory for methamathematics / Smullyan
- Set theory / Kunen
- Incompleteness in the land of sets / Fitting
- Recursively Enumerable Sets and Degres / Soare
- Gödel's theorem / Franzen
- Computability, an introduction to recursive function theory / Cutland
- Logique et théorie des ensembles / Dehornoy
- Gödel's incompleteness theorems / Smullyan
- An introduction to Gödel's theorems / Smith
- Inexhaustibility, a non exhaustive treatment / Franzen

Websites

- http://www.hec.unil.ch/logique/enseignement/recursivity

Moodle Link

- http://moodle.epfl.ch/course/view.php?id=14569