MATH-318 **Set theory**

Duparc Jacques

Cursus	Sem.	Type
Computer science	MA2, MA4	Opt.
Cybersecurity	MA2, MA4	Opt.
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.
SC master EPFL	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Remark

Cours donné en alternance tous les deux ans (donné en 2019-20)

Summary

Set Theory as a foundational system for mathematics. Relative consistency of the Axiom of Choice and the Continuum Hypothesis.

Content

Set Theory: ZFC. Extensionality and Comprehension. Relations, functions, and well-ordering. Ordinals. Class and transfinite recursion. Cardinals. Well-founded relations, Axiom of foundation, induction, and von Neumann's hierarchy. Relativization, absoluteness, reflection theorems. Gödel's constructible universe L. Axiom of Choice, and Continuum Hypothesis inside L. Po-sets, filters and generic extensions. Forcing. ZFC in generic extensions. Cohen Forcing. Independence of the Continuum Hypothesis. HOD and the Axiom of Choice: independence of the Axiom of Choice.

Keywords

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe, L, Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

Learning Prerequisites

Required courses

MATH-381 Mathematical Logic.
In particular ordinal numbers and ordinal arithmetic will be considered known and admitted.

Recommended courses

Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get lost sooner orl later.

Important concepts to start the course

- 1st order logic
- basics of proof theory
- Basics of model theory
- Compacity theorem
- Löwenheim-Skolem

- Completeness theorem

Learning Outcomes

By the end of the course, the student must be able to:

- Specify a model of ZFC
- Prove consistency results
- Develop a generic extension
- Argue by transfinite induction
- Decide whether ZFC proves its own consistency
- Formalize the axioms of ZF, AC, CH, DC
- Sketch an inner model
- Justify the axiom of foundation

Teaching methods

Ex cathedra lecture and exercises

Expected student activities

- Attendance at lectures
- Solve the exercises

Assessment methods

- Writen exam (3 hours)
- Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés

Supervision

Office hours	Yes
Assistants	Yes
Forum	Yes

Resources

Bibliography

1. Thomas Jech: Set theory, Springer 2006
2. Kenneth Kunen: Set theory, Springer, 1983
3. Jean-Louis Krivine: Theorie des ensembles, 2007
4. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
5. Yiannis Moschovakis: Notes on set theory, Springer 2006
6. Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Ressources en bibliothèque

- Introduction to Set theory / Hrbacek

- Set theory / Jech
- Logique et théorie des ensembles / Dehorny
- Set theory / Kunen
- Notes on set theory / Moschovakis
- Theorie des ensembles / Krivine

Websites

- http://www.hec.unil.ch/logique/

Moodle Link

- http://moodle.epfl.ch/course/index.php?categoryid=72