MATH-436 **Homotopical algebra**

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Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

This course will provide an introduction to model category theory, which is an abstract framework for generalizing homotopy theory beyond topological spaces and continuous maps. We will study numerous examples of model categories and their applications in algebra and topology.

Content

1. Category theory
2. Model categories and their homotopy categories
3. Transfer theorems
4. Localizing model categories
5. Monoidal model categories and "brave new algebra"

Keywords

Abstract homotopy theory

Learning Prerequisites

Required courses

Second-year math courses, including Topology.

Recommended courses

• Rings and modules
• Algebraic topology

Important concepts to start the course

• Necessary concept: homotopy of continuous maps
• Recommended concept: chain homotopy of morphisms between chain complexes

Learning Outcomes

By the end of the course, the student must be able to:

• Prove results in category theory involving (co)limits, adjunctions, and Kan extensions

- Prove basic properties of model categories
- Check the model category axioms in important examples
- Apply transfer theorems to establish the existence of model category structures
- Apply Bousfield localization to create model categories with desired weak equivalences
- Compare different model category structures via Quillen pairs
- Transpose results from classical algebra into homotopy-theoretic versions in monoidal model categories
- Check the axioms of a monoidal model category in important cases

Transversal skills

- Demonstrate a capacity for creativity.
- Demonstrate the capacity for critical thinking
- Continue to work through difficulties or initial failure to find optimal solutions.

Teaching methods

Ex-cathedra lectures, exercises

Expected student activities

Handing in weekly exercises to be graded.

Assessment methods

Written exam
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes
Forum	Yes

Resources

Bibliography

- W.G. Dwyer and J. Spalinski, *Homotopy theories and model categories*, Handbook of Algebraic Topology, Elsevier, 1995, 73-126. (Article no. 75 here)

- P.G. Goerss and J.F. Jardine, *Simplicial Homotopy Theory,* Progress in Mathematics **174**, Birkhäuser Verlag, 1999.

- M. Hovey, *Model Categories*, Mathematical Surveys and Monographs **63**, American Mathematical Society, 1999.

- E. Riehl, *Categorical Homotopy Theory,* New Mathematical Monographs **24**, Cambridge University Press, 2014.

Ressources en bibliothèque

- Simplicial Homotopy Theory / Goerss & Jardine
- Categorical Homotopy Theory / Riehl
- Model Categories / Hovey
- (electronic version)
- (electronic version)
- (electronic version)
- Handbook of Algebraic Topology / James
- (electronic version)

Websites

- https://www.epfl.ch/labs/hessbellwald-lab/teaching/2020-2021/

Moodle Link

- http://A link to the course Moodle page will be provided.