

MATH-603

Subconvexity, Periods and Equidistribution

Michel Philippe

Cursus	Sem.	Type
Mathematics		Opt.

Language of teaching	English
Credits	3
Session	
Exam	Oral
Workload	90h
Hours	56
Courses	28
TP	28
Number of positions	20

Frequency

Only this year

Remark

Spring 2021 - Fridays

Summary

This course is a modern exposition of "Duke's Theorems" which describe the distribution of representations of large integers by a fixed ternary quadratic form. It will be the occasion to introduce the students to the adelic language, the theory of automorphic forms and their associated L-functions

Content

Duke's theorems establish that the set of representations of an integer by a fixed ternary quadratic form are usually equidistributed along the level sets hypersurface of the form as the integer becomes large. It has alternative formulations in terms of distribution of Heegner points and their reductions as well as closed geodesics on modular and Shimura curves.

This course will present a modern and unified proof of these theorems by recasting them in the adelic language and by using harmonic analysis on the corresponding adelic locally homogeneous spaces. It will be the occasion to discuss from the adelic viewpoint, the theory of automorphic forms (on PGL_2 and its forms), their periods, their L-functions as well as the subconvexity problem for L-functions.

Course content:

Duke's theorems. Representations by quadratic forms in the classical language.

The local-global principle. The duality principle. Alternative formulations of Duke's Theorems in terms of distribution of Heegner points and closed geodesics.

Recollections on adeles; Adelicization of Duke's theorems.

Automorphic forms on adelic quotients, especially for quaternion algebras.

Automorphic Periods and L-functions: Hecke's and Waldspurger's formula.

The subconvexity problem and its applications to Duke's theorems.

Keywords

L-functions

Automorphic forms

Quadratic forms

Learning Prerequisites

Required courses

Basic course on classical modular forms (cf Iwaniec Book)
Basic course in algebraic number theory (like MATH-482)

Recommended courses

A basic course on the theory of quadratic forms (over \mathbb{Q})
Introduction to analytic number theory (like MATH-313)
MATH-417 (2019) Adelic Number Theory

Teaching methods

Course ex-cathedra

Expected student activities

Participate actively to the course via questions and comments.
To prepare for the lectures by reading in advance the indicated references

Assessment methods

Oral Examination based on the course content

Resources**Bibliography**

Bump, Daniel Automorphic forms and representations. Cambridge Studies in Advanced Mathematics, 55. Cambridge University Press, Cambridge, 1997.
Iwaniec, Henryk Topics in classical automorphic forms. Graduate Studies in Mathematics, 17. American Mathematical Society, Providence, RI, 1997.
O'Meara, O. Timothy Introduction to quadratic forms. Reprint of the 1973 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2000.

Ressources en bibliothèque

- [Automorphic forms and representations / Daniel Bump](#)
- [Topics in classical automorphic forms / Henryk Iwaniec](#)
- [Introduction to quadratic forms / O. Timothy O'Meara](#)

Notes/Handbook

Lecture notes will be distributed as the course progress