EPFL

MATH-459   Numerical methods for conservation laws

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Cursus	Sem.	Type
Computational science and Engineering	MA1, MA3	Opt.
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Introduction to the development, analysis, and application of computational methods for solving conservation laws with an emphasis on finite volume, limiter based schemes, high-order essentially non-oscillatory schemes, and discontinuous Galerkin methods.

Content

The modeling of many problems in the applied sciences and engineering is based on concepts of conservation of mass, momentum and energy, leading to systems of conservation laws. Prominent examples are the Maxwell equations of electromagnetics, the Euler and Navier-Stokes equations of fluid dynamics and equations of elasticity and the systems of magnetohydrodynamics of plasma physics.
In this course we develop, analyse and apply computational methods suitable for solving systems of conservation laws. We begin by discussing fundamental properties of conservation laws, including their ability to generate non-smooth solutions - shocks - from smooth initial conditions, leading to the introduction of weak solutions and entropy conditions. We initiate the discussion of computational methods by finite difference methods for conservation laws and introduce the concept of monotone schemes and the consequences of this. This is followed by a thorough discussion of finite volume methods, including Godunov's methods, monotone fluxes, accuracy, and stability of such methods.
We then begin the discussion of methods of higher order accuracy, first by understanding why higher order accuracy is of interest. This sets the stage for the introduction of limiter based schemes with a particular emphasis on the MUSCL scheme. Subsequently, we discuss in detail essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) methods and their fundamental properties. Higher order in time is achieved through the development of strongly stable Runge-Kutta methods (SSP-RK).
As a final extension to more geneal problems, we develop discontinuous Galerkin methods as a very general and robust high-order accurate extension of finite volume methods. We study these methods in some detail, including their mathematical properties.
Throughout the course there will be an emphasis on mastering mathematical as well as computational aspects of the different methods.

Keywords

Conservation laws, finite volume methods, MUSCL scheme, ENO/WENO methods, discontinuous Galerkin methods

Learning Prerequisites

Required courses

A course in partial differential equations and their numerical approximation. Knowledge of finite difference methods.

Important concepts to start the course

Linear partial differential equations, numerical approximation, stability, convergence. Basic methods for

solving ordinary differential equations and computational linear algebra.

Learning Outcomes

- Choose an appropriate method
- Analyze methods
- Assess / Evaluate computational methods
- Carry out computational experiments
- Carry out mathematical analysis
- Construct computational methods
- Prove basic mathematical properties

Transversal skills

- Assess progress against the plan, and adapt the plan as appropriate.
- Set objectives and design an action plan to reach those objectives.
- Continue to work through difficulties or initial failure to find optimal solutions.
- Take feedback (critique) and respond in an appropriate manner.
- Use both general and domain specific IT resources and tools
- Access and evaluate appropriate sources of information.

Teaching methods

The class will be given as a lecture class with in-class computational experiments to support the analysis.

Expected student activities

Development of computational methods for conservation laws, their analysis, implementation and use for solving application examples of increasing complexity.

Assessment methods

There will be 3 required small reports to be handed in during the class. These will be examined as part of the final oral examination and will count for 50% of the overall grade.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Resources

Bibliography

The class will use the text
J.S. Hesthaven, *Numerical Methods for Conservation Laws: From Analysis to Algorithms.* SIAM Publishing, 2017.

Ressources en bibliothèque

- Numerical Methods for Conservation Laws / Hesthaven

Moodle Link

- https://moodle.epfl.ch/course/view.php?id=14345