MATH-417 **Topics in number theory**

Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Remark

pas donné en 2020-21

Summary

This year's topic is "Adelic Number Theory" or how the language of adeles and ideles and harmonic analysis on the corresponding spaces can be used to revisit classical questions in algebraic number theory

Content

This year we will discuss the theory of adéles and idèles.
Idèles were invented by Claude Chevalley to provide an compact reformulation of Class Field Theory and Artin's reciprocity law.
However the group of idèles together with its associated ring of adèles are powerful tools to encode all sorts of local-global principles in number theory and arithmetic geometry; the associated terminology is by now the lingua franca of the Langlands program.
The course will introduce the language and we will use it to revisit various aspects of classical algebraic number theory. For instance we will give new proofs of classical results like the finiteness of the class group, Dirichlet's units theorem or the class number formula.
-completions in number fields. Ostrowski's Theorem.
-Local-global principles : the case of the space of lattices.
- Topology and harmonic analysis on adeles and ideles.
-The ring of adeles and the group of ideles associated to a number field. Finitness of the class group and Dirichlet's unit theorem all in one.
-Tate's thesis. Analytic properties of Dedekind and Dirichlet L-functions.
- The adelic formulation of class field theory (without proofs)
- Modular forms in the adelic language.

Learning Prerequisites

Required courses

Anbalysis III & IV
Galois Theory.
Rings and Modules.
Introduction to Algebraic Number Theory.

Recommended courses

Not required but possibly useful
Introduction to Analytic Number Theory.
It will be good to have had some exposure to the theory of Riemann's zeta function.

Automorphic forms and L-functions.
Some exposure to the classical theory of modular forms migh be used at the end of the course to compare with the adelic viewpoint.

Important concepts to start the course

-Good knowledge of Galois theory, Algebraic Number Theory and the associated commutative algebra.

Learning Outcomes

By the end of the course, the student must be able to:

- Synthesize the theory of adeles and ideles
- Solve basic problems involving adeles and ideles
- Interpret classical problems in the adelic language

Transversal skills

- Access and evaluate appropriate sources of information.
- Make an oral presentation.
- Demonstrate the capacity for critical thinking

Teaching methods

Ex-Cathedra Course

Expected student activities

We expect a proactive attitude during the courses and the exercises sessions (possibly with individual presentation of the solution of various problems).

Assessment methods

Exam Oral
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes
Forum	No
Others	a moodle with ressources for the course will be maintained

Prerequisite for

Current research in number theory