

MATH-437

Calculus of variations

Stra Federico

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	4 weekly
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Introduction to classical Calculus of Variations and a selection of modern techniques.

Content

- Preliminaries: Hölder functions, Sobolev spaces, functional analysis, convex analysis...
- Model problems: geodesics, brachistochrone, minimal surfaces, isoperimetric problem, Lagrangian mechanics...
- Classical methods: Euler-Lagrange equation, first and second variations...
- Direct methods: coercivity, compactness, lower-semicontinuity...
- Regularity theory for minimizers: Sobolev regularity, Hölder regularity
- Optional if time permits: isoperimetric inequality, Γ -convergence

Keywords

calculus of variations, optimization, minimization, Euler-Lagrange equations, first variation, direct method, Lagrangian, functional analysis, Sobolev spaces, minimal surfaces, convexity, existence, uniqueness, regularity.

Learning Prerequisites**Required courses**

- MATH-200: Analysis III
- MATH-205: Analysis IV
- MATH-303: Measure and integration

Recommended courses

- MATH-301: Ordinary differential equations
- MATH-302: Functional analysis I
- MATH-305: Sobolev spaces and elliptic equations

Important concepts to start the course

The students are required to have sufficient knowledge on real analysis and measure theory. Having taken a course on functional analysis or Sobolev spaces will be an advantage.

Learning Outcomes

By the end of the course, the student must be able to:

- Illustrate historically important optimization problems
- Model geometrical and/or physical problems in the form of optimization
- Analyze the existence and uniqueness of minimizers of optimization problems
- Investigate the regularity properties of minimizers

Teaching methods

Lectures + exercises.

Assessment methods

Oral exam.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Assistants	Yes
Forum	No

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

Main reference:

- *Introduction to the Calculus of Variations*, B. Dacorogna

Other useful resources:

- *Direct Methods in the Calculus of Variations*, E. Giusti
- *Introduction to the Modern Calculus of Variations*, F. Rindler
- *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, H. Brezis
- *Partial Differential Equations*, L. C. Evans

Ressources en bibliothèque

- [Direct Methods in the Calculus of Variations / Giusti](#)
- [Introduction to the Calculus of Variations / Dacorogna](#)
- [Functional Analysis, Sobolev Spaces and Partial Differential Equations / H. Brezis](#)
- [Partial Differential Equations / L. C. Evans](#)