EPFL

MATH-495 **Mathematical quantum mechanics**

Lemm Marius

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	10
Session	Summer
Semester	Spring
Exam	Oral
Workload	300h
Weeks	14
Hours	**6 weekly**
Courses	4 weekly
Exercises	2 weekly
Number of positions	

Summary

Quantum mechanics is one of the most successful physical theories. This course presents the mathematical formalism (functional analysis and spectral theory) that underlies quantum mechanics. It is simultaneously an introduction to mathematical physics and an advanced course in operator theory

Content

Overview of course content.
This course teaches the mathematical framework from functional analysis and spectral theory that grounds quantum mechanics rigorously. The main goals are

- to develop the general theory of the Fourier transform, including tempered distributions
- to develop the theory of unbounded operators on Hilbert spaces, including the spectral theorem for unbounded, self-adjoint operators
- to introduce and investigate spectral types (continuous versus discrete spectrum)
- to connect the mathematics to physics along the way to make concepts from quantum physics precise and rigorous.

Detailed course outline.
In the first week, we will review Banach and Hilbert spaces, especially L^p spaces and Sobolev spaces. We will also introduce the basic ingredients of quantum mechanics: The wave function and the Schr\"odinger equation. Afterwards, the rough plan is as follows.

1. The *first main unit* of the course takes a closer look at the free Schr\"odinger equation. To understand its solutions, we develop the general theory of the Fourier transform (first on L^1, then on L^2, then for tempered distributions). We briefly contrast solutions to the free Schr\"odinger equation with solutions to the heat equation.
2. The *second unit* concerns the general theory of unbounded operators. This part begins with a review of basic Hilbert space theory including the definition of adjoint operator in Hilbert space. The highlight of this unit is the proof of the spectral theorem for unbounded, self-adjoint operators and the associated functional calculus. With the spectral theorem at hand, we define observables and rigorously state the postulates of quantum mechanics.
3. The *third unit* focuses on Schrodinger operators -\Delta+V(x). It is important that these operators are realized in a self-adjoint fashion and so we will discuss various criteria for self-adjointness. Here, it turns out to be beneficial to employ quadratic forms. This unit covers some of the foundational results for Schrodinger operators, such as uniqueness of ground states, the min-max characterization of eigenvalues, and Weyl's theorem that relatively compact perturbations cannot change the essential spectrum. Afterwards, we briefly consider the time-dependent Schrodinger equation. Here, we show what implication the decomposition of the spectral measure into discrete and continuous parts has for quantum dynamics.

These three units form the core material. Depending on time and interest, we may finish with a brief introduction to semiclassical analysis or to many-body quantum mechanics.

Keywords

Mathematical quantum theory; Fourier transform; spectral theory of unbounded operators; quantum observables; wave function; continuous and discrete spectrum; quantum dynamics

Learning Prerequisites

Required courses

Analyse I - IV, Algebre lineaire I et II, Analyse fonctionnelle I

Recommended courses

Analyse I - IV, Algebre lineaire I et II, Analyse fonctionnelle I, Physique quantique I et II

Important concepts to start the course

A firm background in mathematical analysis and rigorous proofs is required. This includes working knowledge of measure theory (especially L^p spaces) and basic functional analysis (especially Hilbert spaces). Familiarity with tools from partial differential equations, in particular Sobolev spaces and the Fourier transform, is not as important, but also useful.
Having previously completed a class on quantum mechanics is very helpful for understanding the motivations behind the results, but not strictly necessary.

Learning Outcomes

By the end of the course, the student must be able to:

- Apply Fourier transform to describe free Schrodinger evolution
- Formulate the basic notions of quantum mechanics rigorously
- Distinguish spectral types and their dynamical implications

Transversal skills

- Communicate effectively with professionals from other disciplines.
- Access and evaluate appropriate sources of information.
- Give feedback (critique) in an appropriate fashion.

Teaching methods

Four hours of lectures, two hours of exercises led by teaching assistant.

Expected student activities

Attend lectures and exercise sessions, read course materials, solve exercises.

Assessment methods

Graded homework sets and oral exam at the end of course.

Supervision

Office hours	Yes
Assistants	Yes
Forum	No

Resources

Bibliography

In addition to the lecture notes mentioned below, the following textbooks are recommended for further reading.

• E.H. Lieb and M. Loss, Analysis, Graduate Studies in Mathematics, Springer, 2001

• M. Reed and B. Simon, Methods of Modern Mathematical Physics: I Functional Analysis, second edition, Academic Press, 1980

• M. Reed and B. Simon, Methods of Modern Mathematical Physics: II Fourier Analysis, Self Adjointness, Academic Press, 1975

Ressources en bibliothèque

• Graduate Studies in Mathematics

• Methods of Modern Mathematical Physics

• vol. 2 Fourier analysis

Notes/Handbook

The course will loosely follow an unpublished set of prior lecture notes that will be available on the course website.