

MATH-483

**Gödel and recursivity**

Cursus	Sem.	Type
Computer science	MA1, MA3	Opt.
Cybersecurity	MA1, MA3	Opt.
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.
SC master EPFL	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Written
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Courses	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Remark**

Cours donné en alternance tous les deux ans (pas donné en 2021-22)

**Summary**

Gödel incompleteness theorems and mathematical foundations of computer science

**Content***Gödel's theorems:*

Peano and Robinson Arithmetics. Representable functions. Arithmetic of syntax. Incompleteness, and undecidability theorems.

*Recursivity :*

Turing Machines and variants. The Church-Turing Thesis. Universal Turing Machine. Undecidable problems (the halting and the Post-Correspondance problems). Reducibility. The arithmetical hierarchy. Relations to Turing machines. Turing degrees.

**Keywords**

Gödel, incompleteness theorems, Peano arithmetic, Robinson arithmetic, decidability, recursively enumerable, arithmetical hierarchy, Turing machine, Turing degrees, jump operator, primitive recursive functions, recursive functions, automata, pushdown automata, regular languages, context-free languages, recursive languages, halting problem, universal Turing machine, Church thesis.

**Learning Prerequisites****Recommended courses**

Mathematical logic (or equivalent). A good understanding of 1st order logic is required - in particular the relation between syntax and semantics.

**Important concepts to start the course**

1st order logic: syntax, semantics, proof theory, completeness theorem, compactness theorem, Löwenheim-Skolem theorem.

**Learning Outcomes**

By the end of the course, the student must be able to:

- Estimate whether a given theory, function, language is recursive or no
- Decide the class that a language belongs to (regular, context-free, recursive,...)

- Elaborate an automaton
- Design a Turing machine
- Formalize a proof in Peano arithmetic
- Sketch the incompleteness theorems
- Propose a non-standard model
- Argue why Hilbert program failed

### Teaching methods

Ex cathedra lecture and exercises

### Assessment methods

Written: 3 hours

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

Office hours	Yes
Assistants	Yes
Forum	Yes

### Resources

#### Virtual desktop infrastructure (VDI)

No

### Bibliography

Set Theory:

- Thomas Jech: Set theory, Springer 2006
- Kenneth Kunen: Set theory, Springer, 1983
- Jean-Louis Krivine: Theory des ensembles, 2007
- Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: <http://www.math.unicaen.fr/~dehornoy/surveys.html>
- Yiannis Moschovakis: Notes on set theory, Springer 2006
- Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Recursion Theory :

- Micheal Sipser: Introduction to the Theory of Computation, Thomson Course Technology Boston, 2006
- Piergiorgio Odifreddi: Classical recursion theory, vol. 1 and 2, Springer, 1999
- Robert I. Soare: Recursively Enumerable Sets and Degrees, A Study of Computable Functions and Computably Generated Sets, Springer-Verlag 1987
- Nigel Cutland: Computability, an introduction to recursive function theory, 1980
- Raymond M. Smullyan: recursion theory for methamathematics, Oxford, 1993

Proof theory :

- Wolfram Pohlers: Proof Theory, the first step into impredicativity, Springer, 2008
- A. S. Troelstra, H. Schwichtenberg, and Anne S. Troelstra: Basic proof theory, Cambridge, 2000
- S.R. Buss: Handbook of proof theory, Springer, 1998

Gödel's results :

- Raymond M. Smullyan: Gödel's incompleteness theorems, Oxford, 1992
- Peter Smith: An introduction to Gödel's theorems, Cambridge, 2008
- Torkel Franzen: Inexhaustibility, a non exhaustive treatment, AK Peteres, 2002
- Melvin Fitting: Incompleteness in the land of sets, King's College, 2007
- Torkel Franzen: Gödel's theorem: an incomplete guide to its use and abuse, AK Peters, 2005

### Ressources en bibliothèque

- [Théorie des ensembles / Krivine](#)
- [Introduction to Set theory / Hrbacek](#)
- [Proof Theory / Pohlers](#)
- [Notes on theory / Moschovakis](#)
- [Basic proof theory / Troelstra](#)
- [Introduction to the Theory of Computation / Sipser](#)
- [Handbook of proof theory / Buss](#)
- [Set theory / Jech](#)
- [Classical recursion theory / Odifreddi](#)
- [Recursion theory for methamathematics / Smullyan](#)
- [Set theory / Kunen](#)
- [Incompleteness in the land of sets / Fitting](#)
- [Recursively Enumerable Sets and Degres / Soare](#)
- [Gödel's theorem / Franzen](#)
- [Computability, an introduction to recursive function theory / Cutland](#)
- [Logique et théorie des ensembles / Dehornoy](#)
- [Gödel's incompleteness theorems / Smullyan](#)
- [An introduction to Gödel's theorems / Smith](#)
- [Inexhaustibility, a non exhaustive treatment / Franzen](#)

### Websites

- <http://www.hec.unil.ch/logique/enseignement/recursivity>

### Moodle Link

- <http://moodle.epfl.ch/course/view.php?id=14569>