MATH-437

# Calculus of variations

| Cursus | Sem. | Type |
|---|---|---|
| Ing.-math | MA2, MA4 | Opt. |
| Mathématicien | MA2 | Opt. |

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| Language of teaching | English |
| Credits | 5 |
| Session | Summer |
| Semester | Spring |
| Exam | Oral |
| Workload | 150h |
| Weeks | 14 |
| **Hours** | **4 weekly** |
| Courses | 2 weekly |
| Exercises | 2 weekly |
| **Number of positions** | |

## Remark

Cours donné en alternance tous les deux ans (pas donné en 2021-22)

## Summary

Introduction to classical Calculus of Variations and a selection of modern techniques.

## Content

- Preliminaries: Hölder functions, Sobolev spaces, functional analysis, convex analysis...
- Model problems: geodesics, brachistochrone, minimal surfaces, isoperimetric problem, Lagrangian mechanics...
- Classical methods: Euler-Lagrange equation, first and second variations...
- Direct methods: coercivity, compactness, lower-semicontinuity...
- Regularity theory for minimizers: Sobolev regularity, Hölder regularity
- Optional if time permits: isoperimetric inequality, $\Gamma$-convergence

## Keywords

calculus of variations, optimization, minimization, Euler-Lagrange equations, first variation, direct method, Lagrangian, functional analysis, Sobolev spaces, minimal surfaces, convexity, existence, uniqueness, regularity.

## Learning Prerequisites

### Required courses

- MATH-200: Analysis III
- MATH-205: Analysis IV
- MATH-303: Measure and integration

### Recommended courses

- MATH-301: Ordinary differential equations
- MATH-302: Functional analysis I
- MATH-305: Sobolev spaces and elliptic equations

### Important concepts to start the course

The students are required to have sufficient knowledge on real analysis and measure theory. Having taken a course on functional analysis or Sobolev spaces will be an advantage.

## Learning Outcomes

By the end of the course, the student must be able to:

- Illustrate historically important optimization problems
- Model geometrical and/or physical problems in the form of optimization
- Analyze the existence and uniqueness of minimizers of optimization problems
- Investigate the regularity properties of minimizers

## Teaching methods

Lectures + exercises.

## Assessment methods

Oral exam.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

## Supervision

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|---|---|
| Assistants | Yes |
| Forum | No |

## Resources

### Virtual desktop infrastructure (VDI)

No

### Bibliography

Main reference:

- *Introduction to the Calculus of Variations*, B. Dacorogna

Other useful resources:

- *Direct Methods in the Calculus of Variations*, E. Giusti
- *Introduction to the Modern Calculus of Variations*, F. Rindler
- *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, H. Brezis
- *Partial Differential Equations*, L. C. Evans

### Ressources en bibliothèque

- Functional Analysis, Sobolev Spaces and Partial Differential Equations / H. Brezis
- Introduction to the Calculus of Variations / Dacorogna
- Direct Methods in the Calculus of Variations /Giusti
- Partial Differential Equations / L. C. Evans