Mathematical foundations of signal processing

Summary
A theoretical and computational framework for signal sampling and approximation is presented from an intuitive geometric point of view. This lecture covers both mathematical and practical aspects of modern signal processing, with hands-on projects, applications and algorithmic aspects.

Content
From Euclid to Hilbert (1/2): Hilbert Spaces and Linear Operators (Vector spaces, Hilbert/Banach spaces; adjoint and inverse operators; projection operators)
From Euclid to Hilbert (2/2): Hilbert Representation Theory (Riesz bases; Gramian; basis expansions; approximations & projections; matrix representations)
Application (1/2): Sampling and Interpolation (Fourier transforms and Fourier series; sampling & interpolation of sequences and functions; Shannon sampling theorem revisited; bandlimited approximation)
Application (2/2): Computerized Tomography (line integrals and projections, Radon transform, Fourier projection/slice theorem, filtered backprojection algorithm).
Regularized Inverse Problems (1/2): Theory (Discrete and functional inverse problems; Tikhonov regularisation; sparse recovery; convex optimisation; representer theorems; Bayesian interpretation)
Regularized Inverse Problems (2/2): Algorithms (Proximal algorithms; gradient descent; primal-dual splitting; computational aspects; numerical experiments and examples)

Learning Prerequisites
Required courses
Signal processing for communications (or Digital signal processing on Coursera)
Linear Algebra I and II (or equivalent).

Recommended courses
Signals and Systems

Important concepts to start the course
Good knowledge of linear algebra concepts. Basics of Fourier analysis and signal processing. Basic
knowledge of Python and its scientific packages (Numpy, Scipy).

Learning Outcomes
By the end of the course, the student must be able to:
• Master the right tools to tackle advanced signal and data processing problems
• Develop an intuitive understanding of signal processing through a geometrical approach
• Get to know the applications that are of interest today
• Learn about topics that are at the forefront of signal processing research
• Identify and implement the algorithm best suited to solve a given convex optimisation problem
• Assess the computational cost and numerical stability of a numerical solver

Transversal skills
• Collect data.
• Write a scientific or technical report.
• Use a work methodology appropriate to the task.
• Demonstrate the capacity for critical thinking
• Use both general and domain specific IT resources and tools

Teaching methods
Ex cathedra with exercises, homeworks and practicals.

Expected student activities
Attending lectures, completing exercises.

Assessment methods
homeworks and project assignment 50%, final exam (written) 50%

Supervision
Office hours No
Assistants Yes
Forum Yes

Resources
Virtual desktop infrastructure (VDI)
No

Bibliography

Ressources en bibliothèque
• Signal Processing: Foundations / Vetterli

Moodle Link