MATH-318 **Set theory**

Duparc Jacques

Cursus	Sem.	Type
Computer science	MA2, MA4	Opt.
Cybersecurity	MA2, MA4	Opt.
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.
SC master EPFL	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Set Theory as a foundational system for mathematics. ZF, ZFC and ZF with atoms. Relative consistency of the Axiom of Choice, the Continuum Hypothesis, the reals as a countable union of countable sets, the existence of a countable family of pairs without any choice function.

Content

Set Theory: ZFC. Extensionality and comprehension. Relations, functions, and well-ordering. Ordinals. Class and transfinite recursion. Cardinals. Well-founded relations, axiom of foundation, induction, and von Neumann's hierarchy. Relativization, absoluteness, reflection theorems. Gödel's constructible universe L. Axiom of Choice (AC), and Continuum Hypothesis inside L. Po-sets, filters and generic extensions. Forcing. ZFC in generic extensions. Cohen Forcing. Independence of the Continuum Hypothesis. HOD and AC: independence of AC. The reals without AC. Symmetric submodels of generic extensions. Applications of the symmetric submodel techinique (the reals as a countable union of countable sets, the reals not well-orderable, every ultrafilter on the integers is trivial). ZF with atoms and permutation models. Simultating permutation models by symmetric submodels of generic extensions.

Keywords

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe, L, Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

Learning Prerequisites

Required courses

MATH-381 Mathematical Logic (or any equivalent course).
In particular ordinal numbers and ordinal arithmetic will be considered known and admitted.

Recommended courses

Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get definitely lost sooner or later.

Important concepts to start the course

- 1st order logic
- basics of proof theory
- Basics of model theory

- Compacity theorem
- Löwenheim-Skolem
- Completeness theorem

Assessment methods

- Writen exam (3 hours)
- Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés

Supervision

Office hours	No
Assistants	Yes
Forum	Yes

Resources

Virtual desktop infrastructure (VDI)

No

Ressources en bibliothèque

- Introduction to Set theory / Hrbacek
- Set theory / Jech
- Theorie des ensembles / Krivine
- Combinatorial Set Theory / Halbeisen
- Notes on set theory / Moschovakis
- Logique et théorie des ensembles / Dehorny
- Set theory / Kunen

Notes/Handbook

Lecture notes on Moodle (397 pages).

Moodle Link

- https://go.epfl.ch/MATH-318