EPFL

MATH-451

Numerical approximation of PDEs

Licht Martin Werner

Cursus	Sem.	Type
Computational science and Engineering	MA2, MA4	Opt.
Financial engineering	MA2, MA4	Opt.
Mathematics	BA6	Opt.
Mechanical engineering	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

The course is about the derivation, theoretical analysis and implementation of the finite element method for the numerical approximation of partial differential equations in one and two space dimensions.

Content

- Linear elliptic problems: weak form, well-posedness, Galerkin approximation.
- Finite element approximation: stability, convergence, a priori error estimates in different norms, implementation aspects.
- Extensions to parabolic and hyperbolic problems

The numerical methods proposed along the class will be implemented during the exercise sessions. The implementation may be in MatLab or python.

Keywords

Partial differential equations, finite element method, Galerkin approximation, stability and convergence analysis.

Learning Prerequisites

Required courses

Analysis I-II-III-IV, Numerical analysis.

Recommended courses

Functional analysis I, Measure and integration, Espaces de Sobolev et équations elliptiques, Advanced numerical analysis, Programming.

Important concepts to start the course

- Basic knowledge of functional analysis: Banach and Hilbert spaces, L^p spaces.
- Some knowledge on theory of PDEs: classical and weak solutions, existence and uniqueness.
- Basic concepts in numerical analysis: stability, convergence, condition number, solution of linear systems, quadrature formulae, finite difference formulae, polynomial interpolation.

Learning Outcomes

By the end of the course, the student must be able to:

- Identify features of a PDE relevant for the selection and performance of a numerical algorithm.
- Assess / Evaluate numerical methods in light of the theoretical results.
- Implement fundamental numerical methods for the solution of PDEs.
- Choose an appropriate discretization scheme to solve a specific PDE.
- Analyze numerical errors and stability properties.
- Interpret results of a computation in the light of theory.
- Prove theoretical properties of discretization schemes.
- State theoretical properties of PDEs and corresponding discretization schemes.
- Analyze numerical errors and stability properties.

Transversal skills

- Use a work methodology appropriate to the task.
- Write a scientific or technical report.
- Use both general and domain specific IT resources and tools

Teaching methods

Ex cathedra lectures, exercises in the classroom and computer lab sessions.

Expected student activities

- Attendance of lectures.
- Completing exercises.
- Solving problems on the computer.

Assessment methods

85% Written exam. The exam may involve the use of a computer.
15% Project involving both computer simulation and theoretical developements.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	Yes
Assistants	Yes
Forum	No

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

- D.F. Griffiths, J.W. Dold, D.J. Silvester. *Essential Partial Differential Equations*. Springer 2015.
- S. Larsson, V. Thomée. *Partial Differential Equations with Numerical Methods.* Springer 2003.
- A. Quarteroni. *Numerical Models for Differential Problems*. Springer 2009.
- S.C. Brenner, L.R. Scott. *The Mathematical Theory of Finite Element Methods*. Springer 2007.

- A. Ern, J-L. Guermond, *Theory and Practice of Finite Elements.* Springer 2004.
- Lecture notes.
- B. Javonic, E. Suli, Anaysis of Finite Difference Schemes, Springer 2014.
- MATLAB documentation from MathWorks.

Ressources en bibliothèque

- Analysis of Finite Difference Scheme / Javonic & Suli
- The Mathematical Theory of Finite Element Methods / Brenner & Scott
- Essential Partial Differential Equations / Griffiths & al.
- MATLAB documentation / Mathwork
- Theory and practice of finite elements / Ern & Guermond
- Numerical Models for Differential Problems / Quarteroni
- Partial Differential Equations with Numerical Methods / Larsson & Thomée

Moodle Link

- https://go.epfl.ch/MATH-451

Prerequisite for

Numerical approximation of PDEs II, Numerical methods for conservation laws, Numerical methods for fluids, structures & electromagnetics