MATH-436

Homotopical algebra

Hess Bellwald Kathryn

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Summary

This course will provide an introduction to model category theory, which is an abstract framework for generalizing homotopy theory beyond topological spaces and continuous maps. We will study numerous examples of model categories and their applications in algebra and topology.

Content

1. Category-theoretic foundations
2. Model categories and their homotopy categories
3. Monoidal model categories

Keywords

Abstract homotopy theory

Learning Prerequisites

Required courses

Second-year math courses, including Topology.

Recommended courses

- Rings and modules
- Algebraic topology

Important concepts to start the course

- Necessary concept: homotopy of continuous maps
- Recommended concept: chain homotopy of morphisms between chain complexes

Learning Outcomes

By the end of the course, the student must be able to:

- Prove results in category theory involving (co)limits, adjunctions, and Kan extensions
- Prove basic properties of model categories
- Check the model category axioms in important examples

- Apply transfer theorems to establish the existence of model category structures
- Apply Bousfield localization to create model categories with desired weak equivalences
- Compare different model category structures via Quillen pairs
- Transpose results from classical algebra into homotopy-theoretic versions in monoidal model categories
- Check the axioms of a monoidal model category in important cases

Transversal skills

- Demonstrate a capacity for creativity.
- Demonstrate the capacity for critical thinking
- Continue to work through difficulties or initial failure to find optimal solutions.

Teaching methods

Flipped class: pre-recorded lectures, active learning sessions with the instructor, exercise sessions with the assistant

Expected student activities

Handing in weekly exercises to be graded.

Assessment methods

Graded exercises
Oral exam
In the case of Article 3 paragraph 5 of the Section Regulations, the teacher decides on the form of the examination he communicates to the students concerned.

Supervision

Office hours	No
Assistants	Yes
Forum	Yes

Resources

Bibliography

- W.G. Dwyer and J. Spalinski, *Homotopy theories and model categories*, Handbook of Algebraic Topology, Elsevier, 1995, 73-126. (Article no. 75 here)

- P.G. Goerss and J.F. Jardine, *Simplicial Homotopy Theory,* Progress in Mathematics **174**, Birkhäuser Verlag, 1999.

- M. Hovey, *Model Categories*, Mathematical Surveys and Monographs **63**, American Mathematical Society, 1999.

- E. Riehl, *Categorical Homotopy Theory,* New Mathematical Monographs **24**, Cambridge University Press, 2014.

Ressources en bibliothèque

- Categorical Homotopy Theory / Riehl
- Model Categories / Hovey
- Simplicial Homotopy Theory / Goerss & Jardine
- (electronic version) Handbook of Algebraic Topology
- (electronic version) Model Categories
- (electronic version) Simplicial Homotopy
- Handbook of Algebraic Topology / James
- (electronic version) Categorical Homotopy Theory

Moodle Link

- https://go.epfl.ch/MATH-436