MATH-473

Complex manifolds

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Contact language	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

Cours pas donné en 22-23, donné en alternance tous les deux ans

Summary

The goal of this course is to help students learn the basic theory of complex manifolds and Hodge theory.

Content

A preliminary outline of the course is as follows.

- Basic concepts of complex manifolds, holomorphic vector bundles, holomorphic tangent and cotangent bundles (for which the corresponding concepts from the real smooth manifolds will be assumed). Canonical line bundles, normal bundle for a submanifold and the adjunction formula.
- Brief description of sheaf cohomology, with deduction of de Rham and Dolbeault cohomology for complex manifolds.
- Hermitian metrics, connections, curvature and Chern classes for complex vector bundles. Case of holomorphic vector bundles.
- Harmonic forms: the Hodge theorem and Serre duality (general results on elliptic operators will be assumed).
- Compact Kaehler manifolds. Hodge and Lefschetz decompositions on cohomology.
- If time allows: Kodaira-Nakano vanishing, Kodaira's embedding theorem.

Learning Prerequisites

Required courses

The students are expected to have already taken a course in complex analysis in one variable and a course in differential and Riemannian geometry.
The course will be aim to be as much as possible of a self-contained treatment of the subject – concepts and basic theorems regarding complex manifolds and complex analysis in many variables will be stated and not assumed.
Some knowledge of algebraic topology (homology, cohomology, characteristic classes) would certainly be useful, as part of the course will be devoted to studying the cohomology of algebraic and related concepts.

Recommended courses

Complex analysis; algebraic topology.
It may be useful if you have already followed a course in algebraic geometry.

Learning Outcomes

By the end of the course, the student must be able to:

- Knowledge of the basic concepts of complex analysis in several variable
- Ability to work with complex structures and the related differential concepts.
- Knowledge of vector bundles and sheaves and ability to compute their cohomology in basic examples.
- Knowledge of the concept of Hodge structure and the ability to work with and manipulate that.

Assessment methods

The final grade will be assigned based on the cumulative points of the student obtained from handed in homework solutions and from the written exam. The weights of the two parts are:
25% - homework
75% - written exam
There will be 4 homeworks that students will be required to hand in on dates to be determined at the start of the course.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Resources

Bibliography

D. Huybrechts, Complex Geometry - an introduction, Springer, 2004.
P. Griffiths and J. Harris, Principles of Algebraic Geometry. Wiley, 1978.
C. Voisin, Complex Geometry and Hodge Theory I. CUP, 2002.

Ressources en bibliothèque

- Principles of algebraic / Griffiths & Harris
- Complex Geometry and Hodge Theory / Voisin
- Complex Geometry / Huybrechts

Moodle Link

- https://go.epfl.ch/MATH-473