EPFL

MATH-530

Introduction to general relativity

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Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Contact language	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Summary

This course will serve as a basic introduction to the mathematical theory of general relativity. We will cover topics including the formalism of Lorentzian geometry, the formulation of the initial value problem for the Einstein equations and applications on the global structure of the spacetime.

Content

General relativity is a theory of gravity in which space time are jointly modeled by a 4 dimensional differentiable manifold equipped with a Lorentzian (i.e. pseudo-Riemannian) metric tensor. The geometry of the spacetime is determined by the matter present at each point via the celebrated Einstein field equations. Therefore, understanding the predictions of general relativity for the structure of our universe requires both a geometric and an analytic approach.
In this course, we will introduce the basic notions of Lorentzian geometry (extending concepts introduced in any course of Riemannian geometry) and study the causal and geometric structure of Lorentzian manifolds appearing as solutions of the Einstein equations. We will also study the initial value problem formulation for the Einstein equations, which will be viewed as a system of evolution equations. Finally, we will explore the consequences of these equations on the global structure of the spacetime in the presence of black holes or in the case of cosmological solutions.
The course will cover the following topics:

- Basic Lorentzian geometry
- Causal structure of Lorentzian manifolds
- The linear wave equation
- The initial value problem for non-linear wave equations
- The Einstein field equations
- Formulation of the initial value problem for the Einstein equations
- Existence and uniqueness of spacetime solutions
- The maximal globally hyperbolic development
- Examples: Black hole solutions and cosmological spacetimes
- Penrose's incompleteness theorem

Keywords

General relativity; Lorentzian geometry; Einstein field equations; non-linear wave equations; initial value problem

Learning Prerequisites

Required courses

The language of the course "Introduction to differentiable manifolds" (MATH-322) will be heavily used. A solid foundation in analysis (including measure theory and functional analysis) will also be necessary. The students should also have attended at least an introductory course in partial differential equations.

Recommended courses

Introductory courses in Riemannian geometry and evolution PDEs would be helpful, but not necessary.

Important concepts to start the course

Differentiable manifold, tensors, partial differential equations

Learning Outcomes

By the end of the course, the student must be able to:

- Use the language of Lorentzian geometry effectively.
- Define the main concepts associated to the initial value problem formulation for the Einstein equations.
- Use the above notions and the basic PDE techniques introduced in the course to solve problems.
- Prove the main theorems appearing in the course.

Transversal skills

- Assess one's own level of skill acquisition, and plan their on-going learning goals.
- Demonstrate a capacity for creativity.
- Demonstrate the capacity for critical thinking
- Access and evaluate appropriate sources of information.

Teaching methods

2h lectures + 2h exercises

Expected student activities

Attending lectures and solve problems sheets; interacting in class; revise course content.

Assessment methods

Final exam.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	Yes
Assistants	No
Forum	No

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

We will follow closely the exposition of the following two books:

Wald, Robert; General relativity, The University of Chicago Press, 1984
Ringström, Hans; The Cauchy problem in General relativity, ESI Lectures in Mathematics & Physics, 2009.

Ressources en bibliothèque

- General relativity / Wald
- The Cauchy problem in General relativity / Ringström

Notes/Handbook

Written notes will be provided.

Moodle Link

- https://go.epfl.ch/MATH-530