EPFL

MATH-561

Spectral theory

Genoud François

Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

This course is an introduction to the spectral theory of linear operators acting in Hilbert spaces. The main goal is the spectral decomposition of unbounded selfadjoint operators. We will also give elementary applications to quantum mechanics.

Content

The first chapter will recall basic properties of Hilbert spaces, linear operators and their spectra.

We will then introduce the notion of integral with respect to a spectral family, which leads naturally to the spectral decomposition of symmetric operators, i.e. bounded selfadjoint operators. This result, based on the Riemann-Stieltjes integration theory, will be our first spectral theorem.

The third chapter will present an approximation procedure which allows one to extend the spectral theorem to unbounded selfadjoint operators, using the Lebesgue-Stieltjes integral. Our approach follows closely the original proof given by Frigyes Riesz in 1952. (If time permits, we will also discuss an alternative proof due to John von Neumann.) Finally, as an important consequence of the spectral theorem, we will prove Stone's theorem describing the structure of one-parameter unitary groups. We will conclude the course with applications to the foundations of quantum mechanics. Notably, the time evolution of a quantum system will be given, via Stone's theorem, by the one-parameter unitary group generated by the Hamiltonian (the "energy") of the system. This leads directly to the Schrödinger equation.

Learning Prerequisites

Required courses

Analysis I-IV; Linear algebra; Functional analysis

Recommended courses

Measure and integration

Learning Outcomes

- Prove properties of bounded and unbounded linear operators in Hilbert spaces
- Solve problems involving symmetric / selfadjoint operators
- Demonstrate a thorough understanding of the spectral decomposition theorem and of Stone's theorem
- Explain how the theory provides the fundamental axioms of quantum mechanics

Assessment methods

Oral exam.

In case art. 3 al. 5 of the Règlement de section applies, the teacher communicates the form of the exam to the concerned students.

Resources

Moodle Link

- https://go.epfl.ch/MATH-561