MATH-487 **Topics in stochastic analysis**

Li Xue-Mei

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	**5 weekly**
Courses	3 weekly
Exercises	2 weekly
Number of positions	

Summary

This course offers an introduction to Markov processes, a widely used model for random evolutions with no memory.

Content

Stochastic Processes describe the evolution of random variables, allowing to include numerous influences, which would not have been possible otherwise. If given its present value, the future value of the process is indepedent of its history, the stochastic process is a Markov process.
The main part of the course will be on discrete processes on a rather general state space, the metric spaces. Markov chains on general state space will be covered to provide motivation and intuition. Time permits we will cover additional material on continuous time Markov processes on an Euclidean space or more generally on a metric space.
This year (Spring 2023) we shall exceptionally include the basic Measure and Integration Theory in the course. In this aspect, the course is largely self-contained. Thereafter, Measure and Integration will be used as foundation of the course. Students are encouraged to read relavant material in advance.

Keywords

Probability, Conditional Expectation, Markov Property, Conditional expectations, Chapman-Kolmogorov equation, Feller Property, Strong Feller property, Kolmogorov's theorem, stopping times, strong Markov property, stationary processes, weak convergence and Prohorov's theorem, invariant measures, Krylov- Bogolubov method, Lyapunov method. Ergodicity by contraction method and Doeblin's criterion. Structures of invariant measures, ergodic theorems. Optional: Diffusion Processes, Markov semigroups and Markov generators, Browniam motions, relation with second order parabolic differential equations, and Brownian motions.

Learning Prerequisites

Required courses

The folowing courses or knowledge on the content of the course will be very helpful: Analysis, Metric and topological spaces, probability, Linear Algebra, Measure and Integration (see comment below concernign the last). Also useful are: ODEs, PDES, and Functional analysis.
The following courses wil be helpful:
Measure ans Integration (Math 303) -- this year I will give a short introduction on this to motivated students who hasnt taken on a course on the Measure and Integration.
Probability Theory (Math 432)
Stochastic Processes (Math 332)

Recommended courses

The courses below are on the pathway of Stochastic Abalysis.

Introduction to stochastic PDEs (Math 485)
Martingales et mouvement brownien (MATH-330)
Stochastic Calculus (Math 431)
Numerical Solutiosn fo Stochastic Differential Equations (Math 450)
Stochastic Simulation (Math 414)
Stochastic epidemic model (Math 560)
Martingales in Mathematical finance (Math 470)

Learning Outcomes

By the end of the course, the student must be able to:

- Demonstrate understanding of the concepts and results from the syllabus includign the proofs of a variey of results
- Apply these concepts and results to tackle a range of problems, including previously unseen ones
- Apply their understanding to develop proofs of unfamiliar results
- Explain thier knowledge of the area in a concise, accurate and coherent manner
- Demonstrate additional competence i nthe subject through the study of more advanced material

Teaching methods

Lectures and Exercise classes

Expected student activities

Attend lectures, problem classes, do exercises and extra reading

Assessment methods

Oral

Supervision

Office hours	No
Assistants	Yes

Resources

Bibliography

-- Stewart N. Ethier and Thomas G. Kurtz. Markov processes.
-- Markov Chains and Mixing Times, by David A. Levin Yuval Peres Elizabeth L. Wilmer
-- Markov Chains, James Norris
-- Markov Chains and stochastic stability, Meyn and Tweedie
-- Bremaud: Markov chains

Ressources en bibliothèque

- Markov processes / Ethier
- Markov Chains and Mixing Times / Levin
- Markov Chains / Norris
- Markov Chains and stochastic stability / Meyn
- Markov chains / Bremaud

Moodle Link

- https://go.epfl.ch/MATH-487