

MATH-451

**Numerical approximation of PDEs**

Buffa Annalisa

Cursus	Sem.	Type
Computational science and Engineering	MA2, MA4	Opt.
Financial engineering	MA2, MA4	Opt.
Mathematics	BA6	Opt.
Mechanical engineering	MA2, MA4	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Lecture	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Summary**

The course is about the derivation, theoretical analysis and implementation of the finite element method for the numerical approximation of partial differential equations in one and two space dimensions.

**Content**

- Linear elliptic problems: weak form, well-posedness, Galerkin approximation.
- Finite element approximation: stability, convergence, a priori error estimates in different norms, implementation aspects.
- Extensions to parabolic and hyperbolic problems

The numerical methods proposed along the class will be implemented during the exercise sessions. The implementation may be in MatLab or python.

**Keywords**

Partial differential equations, finite element method, Galerkin approximation, stability and convergence analysis.

**Learning Prerequisites****Required courses**

Analysis I-II-III-IV, Numerical analysis.

**Recommended courses**

Functional analysis I, Measure and integration, Espaces de Sobolev et équations elliptiques, Advanced numerical analysis, Programming.

**Important concepts to start the course**

- Basic knowledge of functional analysis: Banach and Hilbert spaces,  $L^p$  spaces.
- Some knowledge on theory of PDEs: classical and weak solutions, existence and uniqueness.
- Basic concepts in numerical analysis: stability, convergence, condition number, solution of linear systems, quadrature formulae, finite difference formulae, polynomial interpolation.

**Learning Outcomes**

By the end of the course, the student must be able to:

- Identify features of a PDE relevant for the selection and performance of a numerical algorithm.
- Assess / Evaluate numerical methods in light of the theoretical results.
- Implement fundamental numerical methods for the solution of PDEs.
- Choose an appropriate discretization scheme to solve a specific PDE.
- Analyze numerical errors and stability properties.
- Interpret results of a computation in the light of theory.
- Prove theoretical properties of discretization schemes.
- State theoretical properties of PDEs and corresponding discretization schemes.
- Analyze numerical errors and stability properties.

### Transversal skills

- Use a work methodology appropriate to the task.
- Write a scientific or technical report.
- Use both general and domain specific IT resources and tools

### Teaching methods

Ex cathedra lectures, exercises in the classroom and computer lab sessions.

### Expected student activities

- Attendance of lectures.
- Completing exercises.
- Solving problems on the computer.

### Assessment methods

85% Written exam. The exam may involve the use of a computer.

15% Project involving both computer simulation and theoretical developments.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

Office hours	Yes
Assistants	Yes
Forum	No

### Resources

#### Virtual desktop infrastructure (VDI)

No

#### Bibliography

- D.F. Griffiths, J.W. Dold, D.J. Silvester. *Essential Partial Differential Equations*. Springer 2015.
- S. Larsson, V. Thomée. *Partial Differential Equations with Numerical Methods*. Springer 2003.
- A. Quarteroni. *Numerical Models for Differential Problems*. Springer 2009.
- S.C. Brenner, L.R. Scott. *The Mathematical Theory of Finite Element Methods*. Springer 2007.

- A. Ern, J-L. Guermond, *Theory and Practice of Finite Elements*. Springer 2004.
- Lecture notes.
- B. Javonic, E. Suli, *Analysis of Finite Difference Schemes*, Springer 2014.
- MATLAB documentation from MathWorks.

### Ressources en bibliothèque

- [Analysis of Finite Difference Scheme / Javonic & Suli](#)
- [The Mathematical Theory of Finite Element Methods / Brenner & Scott](#)
- [Essential Partial Differential Equations / Griffiths & al.](#)
- [Partial Differential Equations with Numerical Methods / Larsson & Thomée](#)
- [Theory and practice of finite elements / Ern & Guermond](#)
- [Numerical Models for Differential Problems / Quarteroni](#)
- [MATLAB documentation / Mathwork](#)

### Moodle Link

- <https://go.epfl.ch/MATH-451>

### Prerequisite for

Numerical approximation of PDEs II, Numerical methods for conservation laws, Numerical methods for fluids, structures & electromagnetics