Gödel and recursivity

**Summary**

Gödel incompleteness theorems and mathematical foundations of computer science.

**Content**

**Gödel's theorems:**


**Recursivity:**


**Keywords**

- Gödel, incompleteness theorems, Peano arithmetic, Robinson arithmetic, decidability, recursively enumerable, arithmetical hierarchy, Turing machine, Turing degrees, jump operator, primitive recursive functions, recursive functions, automata, pushdown automata, regular languages, context-free languages, recursive languages, halting problem, universal Turing machine, Church thesis.

**Learning Prerequisites**

**Recommended courses**

- Mathematical logic (or equivalent). A good understanding of 1st order logic is required - in particular the relation between syntax and semantics.

**Important concepts to start the course**

- 1st order logic: syntax, semantics, proof theory, completeness theorem, compactness theorem, Löwenheim-Skolem theorem.

**Learning Outcomes**

By the end of the course, the student must be able to:

- Estimate whether a given theory, function, language is recursive or no
- Decide the class that a language belongs to (regular, context-free, recursive,...)
• Elaborate an automaton
• Design a Turing machine
• Formalize a proof in Peano arithmetic
• Sketch the incompleteness theorems
• Propose a non-standard model
• Argue why Hilbert program failed

Teaching methods
Ex cathedra lecture and exercises

Assessment methods
Written: 3 hours
Dans le cas de l’art. 3 al. 5 du Règlement de section, l’enseignant décide de la forme de l’examen qu’il communique aux étudiants concernés.

Supervision
Office hours Yes
Assistants Yes
Forum Yes

Resources
Virtual desktop infrastructure (VDI)
No

Bibliography
Set Theory:
• Thomas Jech: Set theory, Springer 2006
• Kenneth Kunen: Set theory, Springer, 1983
• Jean-Louis Krivine: Theory des ensembles, 2007
• Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS:
  http://www.math.unicaen.fr/~dehornoy/surveys.html
• Yiannis Moschovakis: Notes on set theory, Springer 2006
• Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Recursion Theory:
• Piergiorgio Odifreddi: Classical recursion theory, vol. 1 and 2, Springer, 1999
• Robert I. Soare: Recursively Enumerable Sets and Degrees, A Study of Computable Functions and
  Computably Generated Sets, Springer-Verlag 1987
• Nigel Cutland: Computability, an introduction to recursive function theory, 1980
• Raymond M. Smullyan: recursion theory for methamathematics, Oxford, 1993

Proof theory:
• Wolfram Pohlers: Proof Theory, the first step into impredicativity, Springer, 2008
• A. S. Troelstra, H. Schwichtenberg, and Anne S. Troelstra: Basic proof theory, Cambridge, 2000

Gödel's results:
• Raymond M. Smullyan: Gödel's incompleteness theorems, Oxford, 1992
• Peter Smith: An introduction to Gödel's theorems, Cambridge, 2008
• Torkel Franzen: Inexhaustibility, a non exhaustive treatment, AK Peteres, 2002
• Melvin Fitting: Incompleteness in the land of sets, King's College, 2007
• Torkel Franzen: Gödel's theorem: an incomplete guide to its use and abuse, AK Peters, 2005

Ressources en bibliothèque
• Théorie des ensembles / Krivine
• Introduction to Set theory / Hrbacek
• Proof Theory / Pohlers
• Notes on theory / Moschovakis
• Basic proof theory / Troelstra
• Introduction to the Theory of Computation / Sipser
• Handbook of proof theory / Buss
• Set theory / Jech
• Classical recursion theory / Odifreddi
• Recursion theory for methamathematics / Smullyan
• Set theory / Kunen
• Incompleteness in the land of sets / Fitting
• Recursively Enumerable Sets and Degres / Soare
• Gödel's theorem / Franzen
• Computability, an introduction to recursive function theory / Cutland
• Logique et théorie des ensembles / Dehornoy
• Gödel's incompleteness theorems / Smullyan
• An introduction to Gödel's theorems / Smith
• Inexhaustibility, a non exhaustive treatment / Franzen

Websites
• http://www.hec.unil.ch/logique/enseignement/recursivity

Moodle Link
• https://go.epfl.ch/MATH-483