EPFL

MATH-410 Riemann surfaces

Mornev Maxim

Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Contact language	English
Credits	5
Session	Winter
Semester	Fall
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Summary

This course is an introduction to the theory of Riemann surfaces. Riemann surfaces naturally appear is mathematics in many different ways: as a result of analytic continuation, as quotients of complex domains under discontinuous group actions, as algebraic curves.

Content

- Category theory
- Homological algebra
- Sheaves and their cohomology
- Complex manifolds
- Topology of compact Riemann surfaces
- Maps between Riemann surfaces. Degree of a map. Riemann-Hurwitz formula
- Differential forms
- De Rham cohomology
- Hodge decomposition
- Holomorphic differentials
- Periods of holomorphic differentials. Jacobian variety
- Abel theorem
- Riemann-Roch theorem
- Embedding of compact Riemann surfaces into projective space
- Compact Riemann surfaces as algebraic curves

Keywords

- Riemann surfaces
- holomorphic maps
- differential forms
- meromorphic functions
- Jacobian variety
- cohomology of sheaves

Learning Prerequisites

Required courses

- Complex analysis
- Differential geometry
- Topology

Recommended courses

- Introduction to differentiable manifolds
- Harmonic analysis

Important concepts to start the course

- Topological spaces
- Manifolds
- Coordinate charts. Change of coordinates
- Differential forms. Integration of differential forms. Stokes theorem
- Holomorphic functions. Cauchy integration formula
- Meromorphic functions. Residue theorem

Learning Outcomes

By the end of the course, the student must be able to:

- Define main mathematical notions introduced in the course
- State main theorems
- Apply main theorems to concrete examples
- Prove main theorems
- Solve problems similar to those discussed on tutorials
- Compute degree of a map, genus of a surface, intersection pairing, period matrix, basis of holomorphic differential forms, image under Abel map, etc.
- Construct examples and counterexamles
- Sketch proves of main results

Transversal skills

- Access and evaluate appropriate sources of information.
- Write a scientific or technical report.
- Demonstrate a capacity for creativity.
- Take feedback (critique) and respond in an appropriate manner.

Teaching methods

- lectures
- tutorials
- feedback on submitted homework solutions

Expected student activities

- attending lectures
- attending tutorials
- submitting written homeworks
- presenting solutions of the exercises

Assessment methods

- midterm home exam 40%
- final exam 60%

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	Yes
Assistants	Yes
Forum	No
Others	Moodle page

Resources

Bibliography

1. S. K. Donaldson. Riemann surfaces
2. J. Jost, Compact Riemann Surfaces: An Introduction to Contemporary Mathematics
3. J. B. Bost, Introduction to Compact Riemann Surfaces, Jacobians, and Abelian Varieties.
4. P. Griffiths and J. Harris, Principles of algebraic geometry.
5. R. Godement. Topologie algébrique et théorie des faisceaux.
6. C. A. Weibel. An introduction to homological algebra.

Ressources en bibliothèque

- Riemann surfaces / Donaldson
- Compact Riemann Surfaces / Jost
- Introduction to Compact Riemann Surfaces, Jacobians, and Abelian Varieties / Bost
- Principles of algebraic geometry / Griffiths & Harris
- Topologie algébrique et théorie des faisceaux / Godement
- An introduction to homological algebra / Weibel

Moodle Link

- https://go.epfl.ch/MATH-410