

MATH-488

**Algebraic K-theory**

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Lecture	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Remark**

pas donné en 2023-24

**Summary**

Algebraic K-theory, which to any ring  $R$  associates a sequence of groups, can be viewed as a theory of linear algebra over an arbitrary ring. We will study in detail the first two of these groups and applications of algebraic K-theory to number theory, algebraic topology, and representation theory.

**Content**

1.  $K_0$  : Grothendieck groups, stability, tensor products, change of rings, the Dévissage, Resolution and Localization theorems and their applications
2.  $K_1$  : elementary matrices, commutators and determinants, long exact sequences relating  $K_0$  and  $K_1$

**Keywords**

Rings and modules, Grothendieck group

**Learning Prerequisites****Required courses**

Second-year algebra and topology courses

**Recommended courses**

Rings and modules (Anneaux et modules)

**Important concepts to start the course**

Elementary ring and field theory

**Learning Outcomes**

By the end of the course, the student must be able to:

- Compute group completions of various semi-groups
- Interpret the universal properties of group completions, Grothendieck groups, and universal determinants
- Compute the Grothendieck group of important subcategories of modules
- Apply the Dévissage, Resolution and Localization theorems

- Sketch the proofs of the Dévissage, Resolution, and Localization theorems
- Explain the functoriality of  $K_0$
- Compare the Grothendieck-type and matrix-based approaches to defining  $K_1$
- Prove elementary properties of  $K_1$

### Transversal skills

- Assess one's own level of skill acquisition, and plan their on-going learning goals.
- Continue to work through difficulties or initial failure to find optimal solutions.
- Demonstrate a capacity for creativity.

### Assessment methods

Each student must hand in one exercise each week for correction, which will determine 30% of the final grade.

The student's performance on the final written exam will determine the other 70% of the grade.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Resources

#### Websites

- <http://gr-he.epfl.ch/AlgKthy20>

#### Moodle Link

- <https://go.epfl.ch/MATH-488>