EPFL

MATH-437	**Calculus of variations**
	Ruf Matthias

Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

donné en alternance tous les deux ans

Summary

Introduction to classical Calculus of Variations and a selection of modern techniques. We focus on inegral functionals defined on Sobolev spaces.

Content

- Preliminaries: Sobolev spaces, weak convergence
- Classical methods: Euler-Lagrange equation and other necessary minimality conditions (with special focus on 1D problems)
- Direct methods: coercivity, lower-semicontinuity, (quasi-)convexity, relaxation, Lavrentiev phenomenon
- If time permits: Gamma-convergence

Keywords

calculus of variations, minimization, integral functionals, Euler-Lagrange equations, variations, direct method, lower semicontinuity, Sobolev spaces, (quasi-)convexity, existence and uniqueness of minimizers.

Learning Prerequisites

Required courses

- MATH-200: Analysis III
- MATH-205: Analysis IV
- MATH-303: Measure and integration

Recommended courses

- MATH-301: Ordinary differential equations
- MATH-302: Functional analysis I
- MATH-305: Sobolev spaces and elliptic equations

Important concepts to start the course

The students are required to have sufficient knowledge on real analysis and measure theory. Having taken a course on functional analysis or Sobolev spaces will be an advantage.

Learning Outcomes

By the end of the course, the student must be able to:
- Discuss the assumptions in a minimization problem
- Apply the direct method of the calculus of variations
- Analyze the existence and uniqueness of minimizers of optimization problems
- Derive the Euler-Lagrange equation and other necessary conditions for minimizers
- Distinguish between scalar and vectorial minimization problems

Teaching methods

Lectures + exercises.

Expected student activities

Following the lectures and solving exercises

Assessment methods

Oral exam.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes
Forum	Yes

Resources

Virtual desktop infrastructure (VDI)
No

Bibliography
Main reference:
- *Introduction to the Calculus of Variations*, B. Dacorogna

Other useful resources:
- *Direct Methods in the Calculus of Variations*, E. Giusti
- *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, H. Brezis
- *Partial Differential Equations*, L. C. Evans

Ressources en bibliothèque
- Functional Analysis, Sobolev Spaces and Partial Differential Equations / H. Brezis
- Direct Methods in the Calculus of Variations /Giusti
- Partial Differential Equations / L. C. Evans
- Introduction to the Calculus of Variations / Dacorogna

Moodle Link

- https://go.epfl.ch/MATH-437