MATH-512

Optimization on manifolds

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	During the semester
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

pas donné en 2023-24

Summary

We develop, analyze and implement numerical algorithms to solve optimization problems of the form: min f(x) where x is a point on a smooth manifold. To this end, we first study differential and Riemannian geometry (with a focus dictated by pragmatic concerns). We also discuss several applications.

Content

- Applications of optimization on manifolds
- First-order Riemannian geometry in Euclidean spaces
- First-order optimization algorithms on manifolds
- Second-order Riemannian geometry in Euclidean spaces
- Second-order optimization algorithms on manifolds
- Fundamentals of differential geometry (general framework)
- Riemannian quotient manifolds (if time permits)
- More advanced geometric tools(if time permits)
- Geodesic convexity(if time permits)

Learning Prerequisites

Required courses

- Analysis
- Linear algebra
- Exposure to numerical linear algebra and numerical methods
- Exposure to optimization (basics such as gradient descent)
- Programming skills in a language suitable for scientific computation (Matlab, Python, Julia...)

There are no prerequisites in differential or Riemannian geometry: we will learn these things together. That said, the course is heavy on proofs, abstract definitions and algorithms. The projects require a substantial amount of work. To complete them, students will need to write nontrivial code, and to develop their own mathematical arguments.

Learning Outcomes

By the end of the course, the student must be able to:

- Manipulate concepts of differential and Riemannian geometry.
- Develop geometric tools to work on new manifolds of interest.
- Recognize and formulate a Riemannian optimization problem.
- Analyze implement and compare several Riemannian optimization algorithms.
- Apply the general theory to particular cases.
- Prove some of the most important theorems studied in class.

Teaching methods

Lectures + exercise sessions

Expected student activities

Students are expected to attend lectures and participate actively in class and exercises. Exercises will include both theoretical work and programming assignments. Students also complete substantial projects (possibly in small groups) that likewise include theoretical and numerical work.

Assessment methods

Projects

Resources

Bibliography

Lecture notes: "An introduction to optimization on smooth manifolds", available online: http://www.nicolasboumal.net/book
- Book: "Optimization algorithms on matrix manifolds", P.-A. Absil, R. Mahoney and R. Sepulchre, Princeton University Press 2008: https://press.princeton.edu/absil
- Book "Introduction to Smooth Manifolds", John M. Lee, Springer 2012: https://link.springer.com/book/10.1007/978-1-4419-9982-5
- Book "Introduction to Riemannian Manifolds", John M. Lee, Springer 2018: https://link.springer.com/book/10.1007/978-3-319-91755-9

Ressources en bibliothèque

- Optimization algorithms on matrix manifolds / P.-A. Absil et al.
- Introduction to Riemannian Manifolds / John M. Lee
- Introduction to Smooth Manifolds / John M. Lee

Moodle Link

- https://go.epfl.ch/MATH-512