

MATH-318

**Set theory**

Duparc Jacques

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Lecture	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Summary**

Set Theory as a foundational system for mathematics. ZF, ZFC and ZF with atoms. Relative consistency of the Axiom of Choice, the Continuum Hypothesis, the reals as a countable union of countable sets, the existence of a countable family of pairs without any choice function.

**Content**

Set Theory: ZFC. Extensionality and comprehension. Relations, functions, and well-ordering. Ordinals. Class and transfinite recursion. Cardinals. Well-founded relations, axiom of foundation, induction, and von Neumann's hierarchy. Relativization, absoluteness, reflection theorems. Gödel's constructible universe  $L$ . Axiom of Choice (AC), and Continuum Hypothesis inside  $L$ . Po-sets, filters and generic extensions. Forcing. ZFC in generic extensions. Cohen Forcing. Independence of the Continuum Hypothesis. HOD and AC: independence of AC. The reals without AC. Symmetric submodels of generic extensions. Applications of the symmetric submodel technique (obtain the reals as a countable union of countable sets, or the reals as not well-orderable, every ultrafilter on the integers is trivial). ZF with atoms and permutation models. Simulating permutation models by symmetric submodels of generic extensions.

**Keywords**

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe,  $L$ , Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

**Learning Prerequisites****Required courses**

MATH-381 Mathematical Logic (or any equivalent course).

In particular ordinal numbers and ordinal arithmetic will be considered known and admitted.

**Recommended courses**

Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get definitely lost sooner or later.

**Important concepts to start the course**

- 1st order logic
- ordinal and cardinal arithmetics
- elements of proof theory

- very basic knowledge of model theory
- the compactness theorem
- Löwenheim-Skolem theorem
- the completeness theorem for 1st order logic

### Learning Outcomes

By the end of the course, the student must be able to:

- Specify a model of ZFC
- Prove consistency results
- Develop a generic extension
- Argue by transfinite induction
- Decide whether ZFC proves its own consistency
- Formalize the axioms of ZF, AC, CH, DC
- Sketch an inner model
- Justify the axiom of foundation
- Formalize a model in which the reals are a countable union of countable sets
- Produce a model in which a countable set of pairs has no choice function
- Create a model in which the finite subsets of an infinite set is mapped onto the set of all its subsets

### Teaching methods

Ex cathedra lecture and exercises

### Expected student activities

- Attendance at lectures
- Solve the exercises

### Assessment methods

- Written exam (3 hours)
- Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés

### Supervision

Office hours	No
Assistants	Yes
Forum	Yes

### Resources

#### Virtual desktop infrastructure (VDI)

No

### Bibliography

1. Kenneth Kunen: Set theory, Springer, 1983
2. Lorenz Halbeisen: Combinatorial Set Theory, Springer 2018
3. Thomas Jech: Set theory, Springer 2006
4. Jean-Louis Krivine: Theorie des ensembles, 2007
5. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS:  
<http://www.math.unicaen.fr/~dehornoy/surveys.html>
6. Yiannis Moschovakis: Notes on set theory, Springer 2006
7. Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

### Ressources en bibliothèque

- [Introduction to Set theory / Hrbacek](#)
- [Set theory / Jech](#)
- [Theorie des ensembles / Krivine](#)
- [Set theory / Kunen](#)
- [Notes on set theory / Moschovakis](#)
- [Logique et théorie des ensembles / Dehornoy](#)
- [Combinatorial Set Theory / Halbeisen](#)

### Notes/Handbook

Lecture notes on Moodle (423 pages).

### Moodle Link

- <https://go.epfl.ch/MATH-318>