

MATH-530

**Differential geometry IV - general relativity**

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Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Lecture	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Summary**

This course will serve as a basic introduction to the mathematical theory of general relativity. We will cover topics including the formalism of Lorentzian geometry, the formulation of the initial value problem for the Einstein equations and applications on the global structure of the spacetime.

**Content**

General relativity is a theory of gravity in which space time are jointly modeled by a 4 dimensional differentiable manifold equipped with a Lorentzian (i.e. pseudo-Riemannian) metric tensor. The geometry of the spacetime is determined by the matter present at each point via the celebrated Einstein field equations. Therefore, understanding the predictions of general relativity for the structure of our universe requires both a geometric and an analytic approach.

In this course, we will introduce the basic notions of Lorentzian geometry (extending concepts introduced in any course of Riemannian geometry) and study the causal and geometric structure of Lorentzian manifolds appearing as solutions of the Einstein equations. We will also study the initial value problem formulation for the Einstein equations, which will be viewed as a system of evolution equations. Finally, we will explore the consequences of these equations on the global structure of the spacetime in the presence of black holes or in the case of cosmological solutions.

The course will cover the following topics:

- Basic Lorentzian geometry
- Causal structure of Lorentzian manifolds
- The linear wave equation
- The initial value problem for non-linear wave equations
- The Einstein field equations
- Formulation of the initial value problem for the Einstein equations
- Existence and uniqueness of spacetime solutions
- The maximal globally hyperbolic development
- Examples: Black hole solutions and cosmological spacetimes
- Penrose's incompleteness theorem

**Keywords**

General relativity; Lorentzian geometry; Einstein field equations; non-linear wave equations; initial value problem

**Learning Prerequisites****Required courses**

The course "Differential Geometry II - Smooth manifolds" is a prerequisite for this course. The student

should also have a solid background in analysis and have attended an introductory course in PDEs.

### Recommended courses

Differential Geometry III - Riemannian geometry.

### Important concepts to start the course

Differentiable manifold, tangent-cotangent space, vector fields, tensors, partial differential equations.

### Learning Outcomes

By the end of the course, the student must be able to:

- Use the language of Lorentzian geometry effectively.
- Define the main concepts associated to the initial value problem formulation for the Einstein equations.
- Use the above notions and the basic PDE techniques introduced in the course to solve problems.
- Prove the main theorems appearing in the course.

### Transversal skills

- Assess one's own level of skill acquisition, and plan their on-going learning goals.
- Demonstrate a capacity for creativity.
- Demonstrate the capacity for critical thinking
- Access and evaluate appropriate sources of information.

### Teaching methods

2h lectures + 2h exercises

### Expected student activities

Attending lectures and solve problems sheets; interacting in class; revise course content.

### Assessment methods

Final exam.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

Office hours	No
Assistants	Yes
Forum	No

### Resources

#### Virtual desktop infrastructure (VDI)

No

### Bibliography

We will follow closely the exposition of the following two books:

Wald, Robert; General relativity, The University of Chicago Press, 1984

Ringström, Hans; The Cauchy problem in General relativity, ESI Lectures in Mathematics & Physics, 2009.

### Ressources en bibliothèque

- General relativity / Wald
- The Cauchy problem in General relativity / Ringström

**Moodle Link**

- <https://go.epfl.ch/MATH-530>