

MATH-417

**Number theory II.b - selected topics**

| Cursus        | Sem.     | Type |
|---------------|----------|------|
| Ing.-math     | MA2, MA4 | Opt. |
| Mathématicien | MA2      | Opt. |

|                            |                 |
|----------------------------|-----------------|
| Language of teaching       | English         |
| Credits                    | 5               |
| Session                    | Summer          |
| Semester                   | Spring          |
| Exam                       | Oral            |
| Workload                   | 150h            |
| Weeks                      | 14              |
| <b>Hours</b>               | <b>4 weekly</b> |
| Lecture                    | 2 weekly        |
| Exercises                  | 2 weekly        |
| <b>Number of positions</b> |                 |

**Remark**

Pas donné en 2024-25. Cours donné en alternance tous les deux ans.

**Summary**

This year's topic is "Adelic Number Theory" or how the language of adèles and ideles and harmonic analysis on the corresponding spaces can be used to revisit classical questions in algebraic number theory.

**Content**

This year we will discuss the theory of adèles and idèles.

Idèles were invented by Claude Chevalley to provide an compact reformulation of Class Field Theory and Artin's reciprocity law.

However the group of idèles together with its associated ring of adèles are powerful tools to encode all sorts of local-global principles in number theory and arithmetic geometry; the associated terminology is by now the lingua franca of the Langlands program.

The course will introduce the language and we will use it to revisit various aspects of classical algebraic number theory. For instance we will give new proofs of classical results like the finiteness of the class group, Dirichlet's units theorem or the class number formula.

-completions in number fields. Ostrowski's Theorem.

-Local-global principles : the case of the space of lattices.

-Topology and harmonic analysis on adèles and ideles (poisson summation formula).

-Adelic points of algebraic groups.

-The ring of adèles and the group of ideles associated to a number field. Finiteness of the class group and Dirichlet's unit theorem all in one.

-Tate's thesis. Analytic properties of Dedekind and Dirichlet L-functions.

- The adelic formulation of class field theory (without proofs)

- Modular forms in the adelic language.

**Keywords**

Local and Global Fields

Archimedean and non-archimedean absolute values

Topological Fields and Rings

Groups of Matrices

L-functions

**Learning Prerequisites****Required courses**

Analysis III & IV  
Introduction to Analytic Number Theory.

### Recommended courses

Not strictly required but certainly useful

- Introduction to Analytic Number Theory.
- Introduction to Algebraic Number Theory.
- Some knowledge of modular forms (such as MATH-511 "Modular forms and applications" ) will be useful since at the end of the course to present modular forms from the adelic viewpoint.

### Important concepts to start the course

Analysis III & IV  
Rings and Modules  
Galois Theory  
Measure and Integration

### Learning Outcomes

By the end of the course, the student must be able to:

- Synthesize the theory of adeles and ideles
- Solve basic problems involving adeles and ideles
- Interpret classical problems in the adelic language
- Solve advanced problems in analytic number theory
- Synthesize the analytic aspects of the theory of numbers

### Transversal skills

- Access and evaluate appropriate sources of information.
- Make an oral presentation.
- Demonstrate the capacity for critical thinking

### Teaching methods

Ex-Cathedra Course

### Expected student activities

We expect a proactive attitude during the courses and the exercises sessions (possibly with individual presentation of the solution of various problems).

### Assessment methods

Oral Exam

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

|              |  |
|--------------|--|
| Office hours | No   |
| Assistants   | Yes  |
| Forum        | No   |
| Others       | a moodle with ressources for the course will be maintained |

### Resources

**Virtual desktop infrastructure (VDI)**

No

**Moodle Link**

- <https://go.epfl.ch/MATH-417>

**Prerequisite for**

Current research in number theory