MATH-487

Topics in stochastic analysis

Li Xue-Mei

Cursus	Sem.	Type
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	6
Session	Summer
Semester	Spring
Exam	Oral
Workload	180h
Weeks	14
Hours	**5 weekly**
Lecture	3 weekly
Exercises	2 weekly
Number of positions	

Summary

This course offers an introduction to topics in stochastic analysis, oriented about theory of multi-scale stochastic dynamics. We shall learn the fundamental ideas, relevant techniques, and in general improve our knowledge of stochastic processes. We touch also trends in current research.

Content

We introduce two-scale systems with slow and fast variables. These variables evolve interactively, but at very different speed. From the standpoint of the slow variable, the fast variables are not tractable. It feels the influence of the fast variable. Multi-scale theory is concerned with iddentifying the effect of the the fast on the slow variable. For two scale interactive slow /fast system of (stochastic) differential equations, we seek an autonomous equation whose solutions approximate the slow variables when the `separation of scale' parameter is large. This theory is strongly linked with ergodicity. Prime examples are Markov processes and solutions of stochastic differential equations. We hope to give an overview of the classical results and touch on recent development and modern techniques.
Motivating models include the evolution of celestial body orbits in an approximate random Hamiltonian system and the approximation of Brownian motions using stochastic processes with a velocity field an Ornstein-Uhlenbeck process, and climat versus weather.

Keywords

Stationary process, ergodicity, Birkhoff's ergodic theorem, Markov processes, invariant measures and ergodicity of Markov processes, Functional large of large numbers for Markov processes, Functional central limit theorems, quantitative theory, and martingales. Special processes such as Ornstein-Uhlenbeck processes and some models involving stochastic differential equations.

Learning Prerequisites

Required courses

Good knowledge of the following are required: Analysis, Probability, Stochastic Processes, Measure and Integration, differential equations (ODE /PDE), Metric spaces and functional analysis. Foundational EPFL courses are: Measure ans Integration (Math 303), Probability Theory (Math 432), Stochastic Processes (Math 332), Martingales et mouvement brownien (MATH-330), Stochastic Calculus (Math 431)

Recommended courses

The courses below are on the pathway of Stochastic Abalysis.
Introduction to stochastic PDEs (Math 485)
Martingales et mouvement brownien (MATH-330)
Stochastic Calculus (Math 431)

Numerical Solutiosn fo Stochastic Differential Equations (Math 450)
Stochastic Simulation (Math 414)
Stochastic epidemic model (Math 560)
Martingales in Mathematical finance (Math 470)

Learning Outcomes

By the end of the course, the student must be able to:

- Apply their understanding to develop proofs of unfamiliar results
- Apply these concepts and results to tackle a range of problems, including previously unseen ones
- Demonstrate additional competence i nthe subject through the study of more advanced material
- Explain thier knowledge of the area in a concise, accurate and coherent manner
- Demonstrate understanding of the concepts and results from the syllabus includign the proofs of a variey of results

Teaching methods

Lectures and Exercise classes

Expected student activities

Attend lectures, problem classes, do exercises and extra reading

Assessment methods

Oral

Supervision

Office hours	No
Assistants	Yes

Resources

Bibliography

-- Stewart N. Ethier and Thomas G. Kurtz. Markov processes.
-- Markov Chains and Mixing Times, by David A. Levin Yuval Peres Elizabeth L. Wilmer
-- Markov Chains, James Norris
-- Markov Chains and stochastic stability, Meyn and Tweedie
-- Bremaud: Markov chains

Ressources en bibliothèque

- Markov chains / Bremaud
- Markov Chains and stochastic stability / Meyn
- Markov processes / Ethier
- Markov Chains and Mixing Times / Levin
- Markov Chains / Norris

Moodle Link

- https://go.epfl.ch/MATH-487