

MATH-437

**Calculus of variations**

Michelat Alexis

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
<b>Hours</b>	<b>4 weekly</b>
Courses	2 weekly
Exercises	2 weekly
<b>Number of positions</b>	

**Remark**

Cours donné en alternance tous les deux ans

**Summary**

Introduction to classical Calculus of Variations and a selection of modern techniques. The Calculus of Variations aims at showing the existence of minimisers (or critical points) of functionals that naturally appear in mathematics and physics (Dirichlet energy, p-energy, etc).

**Content**

- Preliminaries: weak convergence, Sobolev spaces;
- Classical methods: Euler-Lagrange equation and other necessary minimality conditions;
- Direct methods: coercivity, lower-semicontinuity, (quasi-)convexity, relaxation, Lavrentiev phenomenon;
- If time permits: Gamma-convergence.

**Keywords**

Calculus of variations; minimisation; integral functionals; Euler-Lagrange equations; variations; direct method of the calculus of variations; lower semi-continuity; Sobolev spaces; (quasi-)convexity; existence and uniqueness of minimisers.

**Learning Prerequisites****Required courses**

- MATH-200: Analysis III
- MATH-205: Analysis IV
- MATH-303: Measure and integration

**Recommended courses**

- MATH-301: Ordinary differential equations
- MATH-302: Functional analysis I
- MATH-305: Sobolev spaces and elliptic equations

**Important concepts to start the course**

The students are required to have sufficient knowledge on real analysis and measure theory. Having taken a course on functional analysis or Sobolev spaces will be an advantage.

### Learning Outcomes

By the end of the course, the student must be able to:

- Discuss the assumptions in a minimization problem
- Apply the direct method of the calculus of variations
- Analyze the existence and uniqueness of minimizers of optimization problems
- Derive the Euler-Lagrange equation and other necessary conditions for minimizers
- Distinguish between scalar and vectorial minimization problems

### Teaching methods

Lectures + exercises.

### Expected student activities

Following the lectures and solving exercises

### Assessment methods

Oral exam.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

### Supervision

Office hours	No
Assistants	Yes
Forum	Yes

### Resources

#### Virtual desktop infrastructure (VDI)

No

#### Bibliography

Main reference:

- *Introduction to the Calculus of Variations*, B. Dacorogna

Other useful resources:

- *Weak Convergence Methods for Nonlinear Partial Differential Equations*  
L. C. Evans
- *Direct Methods in the Calculus of Variations*, E. Giusti
- *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, H. Brezis
- *Partial Differential Equations*, L. C. Evans

#### Ressources en bibliothèque

- [Functional Analysis, Sobolev Spaces and Partial Differential Equations / H. Brezis](#)
- [Introduction to the Calculus of Variations / Dacorogna](#)
- [Partial Differential Equations / L. C. Evans](#)
- [Direct Methods in the Calculus of Variations /Giusti](#)

**Moodle Link**

- <https://go.epfl.ch/MATH-437>