

MATH-437

Calculus of variations

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Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	4 weekly
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

Cours donné en alternance tous les deux ans

Summary

Introduction to classical Calculus of Variations and a selection of modern techniques. The Calculus of Variations aims at showing the existence of minimisers (or critical points) of functionals that naturally appear in mathematics and physics (Dirichlet energy, p-energy, etc).

Content

- Preliminaries: weak convergence, Sobolev spaces;
- Classical methods: Euler-Lagrange equation and other necessary minimality conditions;
- Direct methods: coercivity, lower-semicontinuity, (quasi-)convexity, relaxation, Lavrentiev phenomenon;
- If time permits: Gamma-convergence.

Keywords

Calculus of variations; minimisation; integral functionals; Euler-Lagrange equations; variations; direct method of the calculus of variations; lower semi-continuity; Sobolev spaces; (quasi-)convexity; existence and uniqueness of minimisers.

Learning Prerequisites**Required courses**

- MATH-200: Analysis III
- MATH-205: Analysis IV
- MATH-303: Measure and integration

Recommended courses

- MATH-301: Ordinary differential equations
- MATH-302: Functional analysis I
- MATH-305: Sobolev spaces and elliptic equations

Important concepts to start the course

The students are required to have sufficient knowledge on real analysis and measure theory. Having taken a course on functional analysis or Sobolev spaces will be an advantage.

Learning Outcomes

By the end of the course, the student must be able to:

- Discuss the assumptions in a minimization problem
- Apply the direct method of the calculus of variations
- Analyze the existence and uniqueness of minimizers of optimization problems
- Derive the Euler-Lagrange equation and other necessary conditions for minimizers
- Distinguish between scalar and vectorial minimization problems

Teaching methods

Lectures + exercises.

Expected student activities

Following the lectures and solving exercises

Assessment methods

Oral exam.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes
Forum	Yes

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

Main reference:

- *Introduction to the Calculus of Variations*, B. Dacorogna

Other useful resources:

- *Weak Convergence Methods for Nonlinear Partial Differential Equations*
L. C. Evans
- *Direct Methods in the Calculus of Variations*, E. Giusti
- *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, H. Brezis
- *Partial Differential Equations*, L. C. Evans

Ressources en bibliothèque

- [Functional Analysis, Sobolev Spaces and Partial Differential Equations / H. Brezis](#)
- [Introduction to the Calculus of Variations / Dacorogna](#)
- [Partial Differential Equations / L. C. Evans](#)
- [Direct Methods in the Calculus of Variations /Giusti](#)

Moodle Link

- <https://go.epfl.ch/MATH-437>