EPFL

MATH-476

Optimal transport

Fernandez-Real Xavier

Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Oral
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

Cours donné en alternance tous les deux ans

Summary

The first part is devoted to Monge and Kantorovitch problems, discussing the existence and the properties of the optimal plan. The second part introduces the Wasserstein distance on measures and develops applications of optimal transport to PDEs, functional/geometric inequalities, traffic models.

Content

The theory of optimal transport began in the eighteenth century with the Monge problem (1781), which is to minimize the cost of transporting an amount of material from the given set of origins to the given set of destinations. In the fourties, Kantorovitch gave an important reformulation of the problem and, since then, the Monge-Kantorovitch problem has been a classical subject in probability theory, economics and optimization. More recently, the interplay between optimal transport and various fields such as PDEs (Ricci flow, Euler equations¿), fluid mechanics, geometric analysis (isoperimetric and Sobolev inequalities, curvature-dimension conditions), functional analysis, urban planning and economics has been deeply investigated.
The first part of the course will be devoted to Monge and Kantorovitch's problems, discussing the existence and the properties of the optimal plan under different conditions on the cost. We will exploit the relation with Kantorovitch's duality theorem, with Brenier's polar decomposition theorem, and with the Monge-Ampere equation, a PDE which arises naturally in this context. The second part of the course will be centered on the applications of optimal transport to different problems: after introducing the Wasserstein distance, we will see the connection with some PDEs and functional/geometric inequalities.

Learning Prerequisites

Required courses

Basic background in analysis (Analysis i-iV, measure theory and metric spaces)

Recommended courses

A few concepts of functional analysis (briefly reviewed along the course).

Learning Outcomes

By the end of the course, the student must be able to:

- Describe the fundamental concepts about Optimal transport, such as the duality theory and the structure of optimal maps
- Solve exercises and master meaningful examples

- Explore and present recent research papers on the topic
- Identify connections between the optimal transport theory and other mathematical problems (such as in PDEs, functional inequalities)

Assessment methods

Oral exam. Exercises presented orally and specific homeworks give a bonus of up to 1.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Resources

Moodle Link

- https://go.epfl.ch/MATH-476