

MATH-417

Number theory II.b - selected topics

Michel Philippe

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Oral
Workload	150h
Weeks	14
Hours	4 weekly
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

Cours donné en alternance tous les deux ans

Summary

This year's topic is "Additive combinatorics and applications." We will introduce various methods from additive combinatorics, establish the sum-product theorem over finite fields and derive various applications (exponential sums, Cayley graphs attached to algebraic groups, etc...).

Content

This year we will discuss various techniques from additive combinatorics mostly in the context of finite fields. After introducing several general techniques and results (Ruzsa calculus, the Balog-Gowers-Szemeredy theorem, ...), we will establish the sum-product phenomenon discovered by Bourgain-Katz-Tao in the context of finite fields and will derive several applications.

These will include:

The study of the mixing properties of certain Cayley graphs for some algebraic group over finite fields due to Helfgott and Bourgain-Gamburd.

Bounds for exponential sums along very small subgroups of the multiplicative group of finite fields due to Bourgain-Gilbichuk-Konyagin.

Keywords

Product sets
 Characters of finite abelian groups
 Arithmetic progressions
 Approximate subgroup
 Ruzsa calculus
 Sum-product phenomenon

Learning Prerequisites**Required courses**

MATH-313: Introduction to Analytic Number Theory.
 MATH-337: Combinatorial number theory

Recommended courses

- Some knowledge of modular forms (such as MATH-511 "Modular forms and applications") may be useful.

Learning Outcomes

By the end of the course, the student must be able to:

- Demonstrate a good mastery of the basics of additive combinatorics
- Solve basic problems of additive combinatorics

Transversal skills

- Access and evaluate appropriate sources of information.
- Make an oral presentation.
- Demonstrate the capacity for critical thinking

Teaching methods

Ex-Cathedra Course

Expected student activities

We expect a proactive attitude during the courses and the exercises sessions (possibly with individual presentation of the solution of various problems).

Assessment methods

Oral Exam

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes
Forum	No
Others	a moodle with ressources for the course will be maintained

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

H. Iwaniec and E. Kowalski: Analytic Number Theory, Colloquium Publ. 53, A.M.S, 2004.

K. Soundararajan: Finite fields, with applications to combinatorics, Student Math. Library 99, American Math. Soc., 2022.

T. Tao and V. Vu: Additive combinatorics, Cambridge Studies in Advanced Math. 105, Cambridge Univ. Press, 2006.

Notes/Handbook

Typed lecture notes will be made available as the course progress.

Prerequisite for

Current research in number theory