MATH-318 **Set theory**

Duparc Jacques

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Set Theory as a foundational system for mathematics. ZF, ZFC and ZF with atoms. Relative consistency of the Axiom of Choice, the Continuum Hypothesis, the reals as a countable union of countable sets, the existence of a countable family of pairs without any choice function.

Content

Set Theory: ZFC. Extensionality and comprehension. Relations, functions, and well-ordering. Ordinals. Class and transfinite recursion. Cardinals. Well-founded relations, axiom of foundation, induction, and von Neumann's hierarchy. Relativization, absoluteness, reflection theorems. Gödel's constructible universe L. Axiom of Choice (AC), and Continuum Hypothesis inside L. Po-sets, filters and generic extensions. Forcing. ZFC in generic extensions. Cohen Forcing. Independence of the Continuum Hypothesis. HOD and AC: independence of AC. The reals without AC. Symmetric submodels of generic extensions. Applications of the symmetric submodel technique (obtain the reals as a countable union of countable sets, or the reals as not well-orderable, every ultrafilter on the integers is trivial). ZF with atoms and permutation models. Simulating permutation models by symmetric submodels of generic extensions.

Keywords

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe, L, Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

Learning Prerequisites

Required courses

MATH-381 Mathematical Logic (or any equivalent course).
In particular ordinal numbers and ordinal arithmetic will be considered known and admitted.

Recommended courses

Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get definitely lost sooner or later.

Important concepts to start the course

- 1st order logic
- ordinal and cardinal arithmetics
- elements of proof theory

- very basic knowledge of model theory
- the compactness theorem
- Löwenheim-Skolem theorem
- the completeness theorem for 1st orderl ogic

Learning Outcomes

By the end of the course, the student must be able to:

- Specify a model of ZFC
- Prove consistency results
- Develop a generic extension
- Argue by transfinite induction
- Decide whether ZFC proves its own consistency
- Formalize the axioms of ZF, AC, CH, DC
- Sketch an inner model
- Justify the axiom of foundation
- Formalize a model in which the reals are a countable union of countable sets
- Produce a model in which a countable set of pairs has no choice function
- Create a model in which the finite subsets of an infinite set is mapped onto the set of all its subsets

Teaching methods

Ex cathedra lecture and exercises

Expected student activities

- Attendance at lectures
- Solve the exercises

Assessment methods

- Writen exam (3 hours)
- Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés

Supervision

Office hours	No
Assistants	Yes
Forum	Yes

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

1. Kenneth Kunen: Set theory, Springer, 1983
2. Lorenz Halbeisen: Combinatorial Set Theory, Springer 2018
3. Thomas Jech: Set theory, Springer 2006
4. Jean-Louis Krivine: Theorie des ensembles, 2007
5. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
6. Yiannis Moschovakis: Notes on set theory, Springer 2006
7. Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Ressources en bibliothèque

- Introduction to Set theory / Hrbacek
- Set theory / Jech
- Theorie des ensembles / Krivine
- Combinatorial Set Theory / Halbeisen
- Notes on set theory / Moschovakis
- Logique et théorie des ensembles / Dehorny
- Set theory / Kunen

Notes/Handbook

Lecture notes on Moodle (423 pages).

Moodle Link

- https://go.epfl.ch/MATH-318