Markov chains and algorithmic applications

Summary
The study of random walks finds many applications in computer science and communications. The goal of the course is to get familiar with the theory of random walks, and to get an overview of some applications of this theory to problems of interest in communications, computer and network science.

Content
Part 1: Markov chains (~6 weeks):
- basic properties: irreducibility, periodicity, recurrence/transience, stationary and limiting distributions,
- ergodic theorem: coupling method
- detailed balance
- convergence rate to the equilibrium, spectral gap, mixing times
- cutoff phenomenon
Part 2: Sampling (~6 weeks)
- classical methods, importance and rejection sampling
- Markov Chain Monte Carlo methods, Metropolis-Hastings algorithm, Glauber dynamics, Gibbs sampling
- applications: function minimization, coloring problem, satisfiability problems, Ising models
- coupling from the past and exact simulation

Keywords
random walks, stationarity, ergodic, convergence, spectral gap, mixing time, sampling, Markov chain Monte Carlo, coupling from the past

Learning Prerequisites
Required courses
Basic probability course
Basic linear algebra and calculus courses

Recommended courses
Stochastic Models for Communications (COM-300)

Important concepts to start the course
Good knowledge of probability and analysis.
Having been exposed to the theory of Markov chains.
Learning Outcomes
By the end of the course, the student must be able to:
• Analyze the behaviour of a random walk
• Assess / Evaluate the performance of an algorithm on a graph
• Implement efficiently various sampling methods

Teaching methods
ex-cathedra course

Expected student activities
active participation to exercise sessions and implementation of a sampling algorithm

Assessment methods
midterm (20%), mini-project (20%), final exam (60%)

Resources

Bibliography
Various references will be given to the students during the course, according to the topics discussed in class.

Ressources en bibliothèque
• Probability and random processes / Grimmett

Notes/Handbook
Lecture notes will be provided

Websites
• https://moodle.epfl.ch/course/view.php?id=15016

Moodle Link
• https://go.epfl.ch/COM-516

Prerequisite for
This course is not so to speak a prerequisite for other courses, but could complement well the course COM-512 on Networks out of control, as well as other courses in statistics.