

MATH-488

Topology IV.a -Algebraic K-theory

Cursus	Sem.	Type
Ing.-math	MA1, MA3	Opt.
Mathématicien	MA1, MA3	Opt.

Language of teaching	English
Credits	5
Session	Winter
Semester	Fall
Exam	Written
Workload	150h
Weeks	14
Hours	4 weekly
Lecture	2 weekly
Exercises	2 weekly
Number of positions	

Remark

pas donné en 2023-24

Summary

Algebraic K-theory, which to any ring R associates a sequence of groups, can be viewed as a theory of linear algebra over an arbitrary ring. We will study in detail the first two of these groups and applications of algebraic K-theory to number theory, algebraic topology, and representation theory.

Content

1. K_0 : Grothendieck groups, stability, tensor products, change of rings, the Dévissage, Resolution and Localization theorems and their applications
2. K_1 : elementary matrices, commutators and determinants, long exact sequences relating K_0 and K_1

Keywords

Rings and modules, Grothendieck group

Learning Prerequisites**Required courses**

Second-year algebra and topology courses

Recommended courses

Rings and modules (Anneaux et modules)

Important concepts to start the course

Elementary ring and field theory

Learning Outcomes

By the end of the course, the student must be able to:

- Compute group completions of various semi-groups
- Interpret the universal properties of group completions, Grothendieck groups, and universal determinants
- Compute the Grothendieck group of important subcategories of modules
- Apply the Dévissage, Resolution and Localization theorems

- Sketch the proofs of the Dévissage, Resolution, and Localization theorems
- Explain the functoriality of K_0
- Compare the Grothendieck-type and matrix-based approaches to defining K_1
- Prove elementary properties of K_1

Transversal skills

- Assess one's own level of skill acquisition, and plan their on-going learning goals.
- Continue to work through difficulties or initial failure to find optimal solutions.
- Demonstrate a capacity for creativity.

Assessment methods

Each student must hand in one exercise each week for correction, which will determine 30% of the final grade.

The student's performance on the final written exam will determine the other 70% of the grade.

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Resources

Websites

- <http://gr-he.epfl.ch/AlgKthy20>

Moodle Link

- <https://go.epfl.ch/MATH-488>