MATH-344

Differential geometry III - Riemannian geometry

Moschidis Georgios

Cursus	Sem.	Type
Mathematics	BA6	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

This course will serve as a first introduction to the geometry of Riemannian manifolds, which form an indispensible tool in the modern fields of differential geometry, analysis and theoretical physics.

Content

Differentiable manifolds appearing in fields ranging from PDEs to theoretical physics usually come equipped with a Riemannian metric; this is simply a symmetric tensor defining an inner product on the space of tangent vectors over each point of the manifold. In contrast to a simple differentiable manifold which only carries topological information, a Riemannian manifold also contains a geometric structure: A Riemannian metric allows us to define notions such as the length of a curve or the distance between two points in the manifold.
In this course, we will introduce the basic concepts associated to Riemannian geometry, such as the Riemannian metric, the curvature tensor and the notion of a geodesic curve. We will then proceed to explore the geometric properties of these objects, in many cases extending ideas and results from Euclidean geometry to this more general setting. We will also discuss the consequences of certain geometric assumptions on the topology of Riemannian manifolds.
The course will cover the following topics:

- Riemannian metrics
- Riemannian connections and geodesics
- The curvature tensors
- The metric structure of a Riemannian manifold and the Hopf-Rinow theorem
- The geometry of hypersurfaces
- Comparison theorems and topological applications

Keywords

Differential geometry; Riemannian metric; Curvature tensor; geodesics

Learning Prerequisites

Required courses

Differential Geometry II - Smooth manifolds, Analysis I-IV.

Important concepts to start the course

Differentiable manifold, tangent-cotangent space, vector fields.

Learning Outcomes

By the end of the course, the student must be able to:

- Define the central objects of Riemannian geometry (Riemannian metric, geodesics, etc)
- Use these objects together with the fundamental identities satisfied by them to solve problems.
- Prove the main theorems appearing in the course.

Transversal skills

- Assess progress against the plan, and adapt the plan as appropriate.
- Demonstrate a capacity for creativity.
- Demonstrate the capacity for critical thinking
- Access and evaluate appropriate sources of information.

Teaching methods

2h lectures + 2h exercises

Assessment methods

Final exam.
Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistants	Yes
Forum	No

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

There are many introductory books on Riemannian geometry, unfortunately most of them intended for an audience of graduate students. We will follow closely the exposition of the following book:
do Carmo, Manfredo; Riemannian geometry. Birkhäuser Boston, Inc., Boston, MA, 1992.
We will sometimes use material from:
Petersen, Peter; Riemannian geometry. Springer-Verlag New York 2006.
Both manuscripts are available at the EPFL library.
Ressources en bibliothèque

- Riemannian geometry / do Carmo
- Riemannian geometry / Petersen

Ressources en bibliothèque

- Find the references at the Library

Moodle Link

- https://go.epfl.ch/MATH-344

Prerequisite for

Differential Geometry IV - General Relativity